

Numerical Methods

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VALLIAMMAI ENGINEERING COLLEGE
MA6459 - NUMERICAL METHODS
DEPARTMENT OF MATHEMATICS
QUESTION BANK
UNIT - I SOLUTION OF EQUATIONS
PART- A

1. Give an example of transcendental and algebraic equation?
2. What are the merits of Newton's method of iterations.
3. State the condition & order for convergence of N – R method.
4. Find an iterative formula for (i) N^P (ii) $1/N$, where N is a positive number by using Newton – Raphson method.
5. On what type of equations Newton's method can be applicable.
6. State the General Newton Raphson Method.
7. What is the Criterion for the convergence of Newton's – Raphson method.
8. State fixed point theorem and the fixed point iteration formula.
9. What are the advantages of iterative methods over direct method of solving a system of linear algebraic equations.
10. Can we apply iteration method to find the root of the equation $2x - \cos x = 5$ in $\left[0, \frac{\pi}{2}\right]$?
11. Find the positive root of $x^2 + 5x - 3 = 0$ using fixed point iteration method starting with 0.6 as first approximation
12. Find the root of $xe^x - 3 = 0$ in $1 < x < 1.1$ by Iteration method.
13. State the condition for Convergence of Iteration method.
14. Find inverse of $A = \begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix}$ Power method.
15. Find the first iteration values of x, y, z by Gauss seidel method
 $28x + 4y - z = 32; x + 3y + 10z = 24; 2x + 17y + 4z = 35$
16. State the condition for the convergence of Gauss Seidel iteration method for solving a system of linear equation
17. By Gauss Elimination method solve (i) $x + y = 2$ and $2x + 3y = 5$ (ii) $2x - y = 1, x - 3y + 2 = 0$
(iii) $x - 2y = 0, 2x + y = 5$
18. Give two direct method to solve system of linear equations.
19. Compare Gauss Elimination, Gauss Jordan method.
20. Compare Gauss seidel method, Gauss Jordan method.

PART –B

1. (i) Find the positive real root of $3x - \cos x - 1 = 0$ using Newton – Raphson method.

(ii) Find the dominant eigen value and vector of $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ using Power method.

2. (i) Find the positive real root of $2x - \log_{10} x - 6 = 0$ using Newton – Raphson method.

(ii) Find the dominant eigen value and vector of $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 \\ -4 & 7 \\ 0 & 0 \end{pmatrix}$ using Power method.

3. (i) Find the positive real root of $x^3 - 5x + 3 = 0$ using Iteration method.

(ii) Find the inverse of the matrix $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$ using Gauss Jordan method.

4. (i) Find the positive real root of $2x^3 - 3x - 6 = 0$ using Iteration method.

(ii) Find the inverse of the matrix $\begin{pmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{pmatrix}$ using Gauss Jordan method.

5. (i) Find the iterative formula to find N where N is positive integer using

Newton's method and hence find 142 .

- (ii) Solve by Gauss Elimination method $x + 5y + z = 14$; $2x + y + 3z = 13$; $3x + y + 4z = 17$

6. (i) Find the iterative formula to find N where N is positive integer using

Newton's method and hence find 11 .

- (ii) Solve by Gauss Elimination method $10x + y + z = 12$; $2x + 10y + z = 13$; $x + y + 5z = 7$.

7. (i) Find the inverse of the matrix $\begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ using Gauss Jordan method.

- (ii) Solve by Gauss Jordan method $3x + 4y + 5z = 18$; $2x - y + 8z = 13$; $5x - 2y + 7z = 20$

8. (i) Find the inverse of the matrix $\begin{pmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{pmatrix}$ using Gauss Jordan method.

- (ii) Apply Gauss seidel method to solve system of equations $30x - 2y + 3z = 75$;
 $2x + 2y + 18z = 30$; $x + 17y - 2z = 48$

9. (i) Find the dominant eigen value and vector of $A = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$ using Power method.

(ii) Apply Gauss seidel method to solve system of equations $28x + 4y - z = 32$;
 $x + 3y + 10z = 24$; $2x + 17y + 4z = 35$

10. (i) Find the inverse of the matrix $\begin{pmatrix} 1 & 2 & -1 \\ 1 & 0 \\ 4 & -1 & 3 \\ 2 \end{pmatrix}$ using Gauss Jordan method.

(ii) Apply Gauss seidel method to solve system of equations $x + y + 54z = 110$;
 $27x + 6y - z = 85$; $6x + 15y - 2z = 72$

UNIT - II INTERPOLATION AND APPROXIMATION

PART -A

1. State Lagrange's interpolation formula for unequal intervals.
2. Using Lagrange's find a polynomial through (0,0) (1,1) and (2,2).
3. Give inverse Lagrange's interpolation formula.
4. Write the Lagrange's formula for y, if three sets of values are given?
5. Find the divided differences with arguments a, b, c if $f(x) = 1/x$
6. Find the divided differences of for the arguments 1, 3, 6, 11.
7. Form the divided difference table for the following data (0,1), (1,4), (3,40) and (4,85)
8. Find the interpolating polynomial for the given data:

x:	2	5	7	8
f(x):	1	2	3	4
9. , find f(a,b) and f(a,b,c) using divided differences .

10. Construct a divided difference table for the following data

x:	4	4	7	10	11	13
f(x):	48	100	294	900	1210	2028

11. State Newton's forward interpolation formula.
12. When should we use Newton's backward difference formula?
13. Define a cubic spline S(x) which is commonly used for interpolation
14. State the properties of cubic spline.
15. Supply the condition for a spline to be cubic.
16. Find y when x = 0.5 given x :

x:	0	1	2
y:	2	3	12

17. Find $y(0.5)$ given

x :	0	1	4
y:	4	3	24

18. State Newton's forward formula upto 3rd finite differences.

19. What is the nature of the n^{th} divided differences of a polynomial.

20. Show that the divided differences are symmetrical in their arguments.

PART –B

1.(i). Using Newton's forward interpolation formula find the value of 1995 from the following table

x:	1951	1961	1871	1981
y:	35	42	58	84

(ii) Using Newton's divided difference formula From the following table ,find $f(8)$

X:	3	7	9	10
F(x):	168	120	72	63

2.(i) Using Newton's forward interpolation formula find the value of $y(46)$ from the following

X:	45	50	55	60	65
Y:	114.84	96.16	83.32	74.48	68.48

(ii) Using Newton's divided difference formula From the following table , Find $f(9)$ from the following

x:	5	7	11	13	17
y:	150	392	1452	2366	5202

3. (i) Using Newton's backward formula find the 6th term of sequence 8, 12, 19, 29, 42 .

(ii) From the following table find the number of students who obtain marks between 40 and 45

Marks :	30 - 40	40 -50	50 – 60	60 – 70	70 - 80
No of stu :	31	42	51	35	31

4. (i) Using Newton's backward formula find $f(7.5)$ from the following table

X :	1	2	3	4	5	6	7	8
Y: 1	8	27	64	125	216	343	512	

(ii) Find $f(2)$, $f(8)$ and $f(15)$ from the following using Newton's divided difference formula

x:	4	5	7	10	11	13
y:	48	100	294	900	1210	2028

5. (i) Find the natural spline

x:	0	1	2	3
y:	1	4	0	-2

(ii) Using Lagrange's method Find x when $y = 20$ from the following

x:	1	2	3	4
y:	1	8	27	64

6. (i) Using Lagrange's method, Find the polynomial $f(x)$ and hence find $f(5)$

x:	1	3	4	6
y:	-3	0	30	132

(ii) Obtain Cubic Spline to the following data

x:	-1	0	1	2
y:	-1	1	3	35

given $y_0 = y_3 = 0$.

7. (i) From the following data taken from steam table Find the pressure $t = 142$ and $t = 175$.

Temp :	140	150	160	170	180
Pressure:	3.685	4.854	6.302	8.076	10.225

- (ii) Fit a polynomial using Inverse Lagrange's method
- | | | | | |
|----|---|---|---|---|
| x: | 1 | 2 | 7 | 8 |
| y: | 1 | 5 | 5 | 4 |

8. (i) Find $y(0.5)$ and $y'(1)$ given that $M_0 = M_2 = 0$ using Cubic Spline

x:	0	1	2
y:	-5	-4	3

- (ii) Find the polynomial of degree 3 from the following using Newton's formula

X:	0	1	2	3	4	5	6	7
Y:	1	2	4	7	11	16	22	29

9. (i) Find $y(43)$ from the following

X:	40	50	60	70	80	90
Y:	184	204	226	250	276	304

- (ii) Using Cubic Spline to the following data, Find $y(1.5)$

x:	1	2	3	4
y:	1	2	5	11

- 10.(i) Find the polynomial and hence find $f(1)$ using Lagrange's method

x:	-1	0	2	3
y:	-8	3	1	12

- (ii) Find the spline interpolation
- | | | | | | |
|----|---|---|---|---|---|
| x: | 1 | 2 | 3 | 4 | 5 |
| y: | 1 | 0 | 1 | 0 | 1 |

UNIT – III NUMERICAL DIFFERENTIATION AND INTEGRATION

1. State Newton's backward difference formula to find $\left(\frac{dy}{dx}\right)_{x=x_n}$ and $\left(\frac{d^2y}{dx^2}\right)_{x=x_n}$
2. What do you mean by numerical differentiation.
3. Show that the divided difference operator is linear
4. Why is Trapezoidal rule so called?
5. State the formula for trapezoidal rule of integration.
6. Write Simpson's 3/8 rule, assuming $3n$ intervals.
7. State Simpson's one third rule.
8. In numerical integration, what should be the number of intervals to apply Simpson's one – third rule and Simpson's three – eighths rule.
9. Compare trapezoidal rule and Simpson's one third rule.
10. Write down the order of the errors of Simpson's one third rule.
11. Write down the order of the errors of trapezoidal rule.
12. Compare Simpson's 3/8 rule and Simpson's one third rule.
13. State the formula for 2 – point Gaussian quadrature.

14. Using two point Gaussian quadrature formula , evaluate $\int_{-1}^1 \frac{1}{1+x^4} dx$
15. Using two point Gaussian quadrature formula , evaluate $\int_{-1}^1 \frac{1}{1+x^2} dx$
16. Write down the abscissa and weights of the three point Gaussian integration.
17. Apply Simpson's 1/3rd rule to find $I = \int_0^1 1-x^2 dx$ taking $h = 0.1$
18. State Romberg's integration formula to find the value of $I = \int_a^b f(x)dx$ using h & $h/2$.
19. Evaluate $\int_1^4 f(x)dx$ from the table by Simpson's 3/8th rule
- | | | | | |
|----------|---|---|----|----|
| x : | 1 | 2 | 3 | 4 |
| $f(x)$: | 1 | 8 | 27 | 64 |
20. Find $f'(x)$ using Newton's divided difference method.

PART -B

- 1.(i) A Jet fighters position on an air craft carries runway was timed during landing
- | | | | | | | | |
|-------------------|-------|-------|-------|-------|-------|-------|--------|
| t, sec : | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 |
| y, m : | 7.989 | 8.403 | 8.781 | 9.129 | 9.451 | 9.750 | 10.031 |
- where y is the distance from end of carrier the velocity and acceleration at $t = 1.0$, $t = 1.6$
- (ii) By dividing the range into 10 equal parts , evaluate $\int_0^\pi \sin x dx$ using Simpson's 1/3 rule.
- 2.(i) Obtain first and second derivative of y at $x = 0.96$ from the data
- | | | | | | |
|-------|--------|--------|--------|--------|--------|
| x : | 0.96 | 0.98 | 1 | 1.02 | 1.04 |
| y : | 0.7825 | 0.7739 | 0.7651 | 0.7563 | 0.7473 |
- (ii) Evaluate $\int_0^1 \frac{1}{1+x^2} dx$, using trapezoidal rule and Simpson's 3/8th rules.
- 3.(i) The table given below reveals the velocity of the body during the time t specified .Find its acceleration at $t=1.1$
- | | | | | | |
|-------|------|------|------|------|------|
| t : | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 |
| v : | 43.1 | 47.7 | 52.1 | 56.4 | 60.8 |
- (ii) Evaluate $\int_2^{4.4} \int_2^{4.4} xy dx dy$ using Simpson's 1/3rd rule, divide the range into 4 equal parts.
- 4.(i) Using the given data find $f'(5)$ and $f'(6)$
- | | | | | | | |
|----------|---|----|----|-----|-----|-----|
| x : | 0 | 2 | 3 | 4 | 7 | 9 |
| $f(x)$: | 4 | 26 | 58 | 112 | 466 | 992 |

(ii) Evaluate (i) $\int_1^2 \int_1^2 \frac{dx dy}{x+y}$ $h = k = 0.25$ using trapezoidal, Simpson's rule .

5. (i) From the following table find the value of x for which f (x) is maximum

x :	60	75	90	105	120
f(x):	28.2	38.2	43.2	40.9	37.7

(ii) Apply Gaussian three point formula to find $\int_{0.2}^{1.5} e^{-r^2} dr$

6.(i) Find the value of f ' (8) from the table given below

x :	6	7	9	12
f(x) :	1.556	1.690	1.908	2.158

(ii) Evaluate $\int_1^{1.4} \int_1^{2.4} xy dx dy$ using trapezoidal, Simpson's rule .

7. (i) Evaluate $\int_0^1 \int_0^1 (1+x+y) dx dy$ using trapezoidal, Simpson's rule .

(ii) Calculate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 3.5 from the data given below by Newton's backward

difference formula x :	1.5	2	2.5	3	3.5	4
y :	3.375	7	13.625	24	38.875	59

8.(i) Obtain f ' (0.02) and f'' (0.05)

x :	0.01	0.02	0.03	0.04	0.05	0.06
y :	0.1023	0.1047	0.1071	0.1096	0.1122	0.1148

(ii) Apply Gaussian three point formula to find $\int_0^1 \frac{dt}{1+t}$.

9.(i) Given that x : 1.1 1.2 1.3 1.4 1.5

y :	8.403	8.781	9.129	9.451	9.75
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find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 1.1

(ii) Apply Gaussian three point formula to find $\int_3^7 \frac{dx}{1+x^2}$

10.(i) A river is 80 meter wide the depth d in meters at a distance x meters from one bank is given below. Calculate the area of the cross section of the river using Simpson rule

x :	0	10	20	30	40	50	60	70	80
d :	0	4	7	9	12	15	14	8	3

(ii) Find the first second derivative of the function f (x) = x³ - 9x -14 at x = 3.0 using the values given below

x :	3	3.2	3.4	3.6	3.8	4
f(x) :	-14	-10.03	-5.296	-0.256	-6.672	14

UNIT – IV INITIAL VALUE PROBLEMS FOR ORDINARY DIFFERENTIAL EQUATIONS

PART-A

- 1 Define initial value problems
2. Explain the terms initial and final value problems
3. State the Taylor series formula to find $y(x_1)$ for solving $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$
4. Find Taylor's series upto x^3 terms satisfying
5. Solve numerically $\frac{dy}{dx} = x + y$ when using Taylor series upto with
6. State Euler's iteration formula for ordinary differential equation
7. Using Euler's method, find if $\frac{dy}{dx} = x^2 + y^2$, taking
8. Using Euler's method find given that
9. Using Euler's method, solve $\frac{dy}{dx} = y - x^2$ given, for. Take
10. Using Euler's method find y at $x = 0.2, 0.4, 0.6$ given that $\frac{dy}{dx} = 1 - 2xy$ $y(0) = 0$ with $h = 0.2$.
11. State the Euler's modified formula for solving $\frac{dy}{dx} = f(x, y), y(x_0) = y_0$
12. Use modified Euler's method to find $y(0.4)$ given
13. Apply fourth order Runge – Kutta method to find $y(0.1)$ given $\frac{dy}{dx} = x + y$ $y(0) = 1, h = 0.1$
14. Write down the Runge – Kutta method of order 4 for solving initial value problems in ordinary differential equation
15. Use R-K method of second order to find $y(0.4)$ given
- 16 Explain one step methods and multi step methods
17. State Adam's predictor corrector formula
18. Write Milne's predictor corrector formula
19. What is predictor corrector method?
20. Mention the multi step methods available for solving ordinary differential equation

PART- B

- 1.(i) Solve $\frac{dy}{dx} = \log_{10}(x + y)$ $y(0) = 2$ By Euler modified method to find the values of $y(0.2)$ $y(0.4)$ and $y(0.6)$ by taking $h = 0.2$.

(ii) Using Runge-kutta method of 4th solve the following equation taking each step $h = 0.1$

$$\text{for } \frac{dy}{dx} = \left[\frac{4t}{y} - t \cdot y \right] \text{ given } y(0) = 3 \text{ calculate } y \text{ at } x = 0.1 \text{ and } 0.2.$$

2.(i) Using Taylor series method find y at $x = 0.1$ given $\frac{dy}{dx} = 2y + 3e^x$ $y(0) = 0$.

(ii) Solve $2y' - x - y = 0$ given $y(0) = 2$, $y(0.5) = 2.636$, $y(1) = 3.595$, $y(1.5) = 4.968$ to get $y(2)$ by Adam's method.

3.(i) Using Taylor series method find correct to 4 decimal places the value of $y(0.1)$ given

$$\frac{dy}{dx} = x^2 + y^2 \quad y(0) = 1.$$

(ii) Obtain $y(0.6)$ given $\frac{dy}{dx} = x + y$, $y(0) = 1$ using $h = 0.2$ by Adam's method if

$$y(-0.2) = .8373, y(0.2) = 1.2427, \text{ and } y(0.4) = 1.5834.$$

4.(i) Using Euler's method, solve numerically the equation $y' = x + y$, $y(0) = 1$ for $x = 0.0$ (0.2) (1.0). Check your answer with the exact solution.

(ii) Solve and find $y(2)$ by Milne's method $\frac{dy}{dx} = \frac{1}{2}(x + y)$, given $y(0) = 2$, $y(0.5) = 2.636$,

$$y(1.0) = 3.595 \text{ and } y(1.5) = 4.968.$$

5.(i) Using modified Euler methods find $y(0.1)$ and $y(0.2)$ given $\frac{dy}{dx} = x^2 + y^2$ $y(0) = 1$ and $h = 0.1$

(ii) Given $\frac{dy}{dx} = x^2(1 + y)$ $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$, evaluate $y(1.4)$

By Adam's Bashforth predictor corrector method

6.(i) Apply modified Euler method to find $y(0.2)$ and $y(0.4)$ given $y' = x^2 + y^2$, $y(0) = 1$, $h = 0.2$.

(ii) Use Milne's method find $y(0.4)$ given $\frac{dy}{dx} = xy + y^2$ $y(0) = 1$, using Taylor series method

find $y(0.1)$, $y(0.2)$ and $y(0.3)$

7.(i) Using Runge - kutta method of order 4 solve $y'' - 2y' + 2y = e^{2x} \sin x$, with $y(0) = -0.4$, $y'(0) = -0.6$ to find $y(0.2)$.

(ii) Given $\frac{dy}{dx} = \frac{1}{2}(1 + x^2)y^2$ and $y(0) = 1$, $y(0.1) = 1.06$, $y(0.2) = 1.12$, $y(0.3) = 1.21$, evaluate $y(0.4)$

by Milne's predictor - corrector method.

8.(i) Solve $\frac{dy}{dx} = y - x^2$, $y(0) = 1$. Find $y(0.1)$ and $y(0.2)$ by R-K method of fourth order, find $y(0.3)$

by Euler's method, find $y(0.4)$ by Milne's method.

(ii) Using Milne's method find $y(4.4)$ given $5xy' + y^2 - 2 = 0$ given $y(4) = 1$; $y(4.1) = 1.0049$; $y(4.2) = 1.0097$; and $y(4.3) = 1.0143$.

9. (i) Using Taylor series method find the value of y at $x = 0.1$ and 0.2 to 4 decimal places given

$$\frac{dy}{dx} = x^2 y - 1 \quad y(0) = 1.$$

(ii) Given $\frac{dy}{dx} = xy + y^2$ $y(0)=1, y(0.1) = 1.1169, y(0.2) = 1.2773, y(0.3) = 1.5049$, evaluate $y(0.4)$

by Milne's method

10(i) Using Runge-kutta method of 4th solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ given that $y(0) = 1$ at $x = 0.2$ and 0.4

(ii) Given $\frac{dy}{dx} = xy + y^2$ $y(0)=1, y(0.1) = 1.1169, y(0.2) = 1.2773, y(0.3) = 1.5049$,

evaluate $y(0.4)$ by Milne's method.

UNIT – V BOUNDARY VALUE PROBLEMS IN ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS

PART-A

1. Define difference quotient of a function $y(x)$
2. State the finite difference approximation for $\frac{d^2y}{dx^2}$ and state the order of truncation error
3. Write down the finite difference scheme for the differential equation $\frac{d^2y}{dx^2} - 3y = 2$
4. Obtain the finite difference scheme for the differential equation $2 \frac{d^2y}{dx^2} + y = 5$
5. Express $y''(x) + a(x)y'(x) + b(x)y(x) = f(x)$ in terms of difference quotients
6. Write standard five point formula and diagonal five point formula used in solving Laplace equation $U_{xx} + U_{yy} = 0$ at the point $(i\Delta x, j\Delta y)$
7. Write down Laplace's equation and its finite difference analogue and the standard five point formula
8. Write down the diagonal five point formula in Laplace equation
9. Write down the two dimensional Laplace's equation and Poisson's equation
10. Write down Poisson's equation and its finite difference analogue
11. What is the order of error in solving Laplace and Poisson's equation by using finite difference method?
12. Write down the finite difference scheme for solving the Poisson's equation
13. State one dimensional heat equation and its boundary conditions
14. Name at least two numerical methods that are used to solve one dimensional diffusion equation
15. State the implicit finite difference scheme for one dimensional heat equation
16. What is Bender – Schmidt recurrence equation? For what purpose it is used?
17. Write down one dimensional wave equation and its boundary conditions
18. State the explicit formula for the one dimensional wave equation with $1 - \lambda^2 a^2 = 0$ where $\lambda = k/h$ and $a^2 = T/m$

19. State explicit finite difference scheme for one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$

20. Derive Crank – Nicholson scheme

PART-B

1(i) Solve the boundary value problem $x^2 y'' - 2y + x = 0$ subject to $y(2) = 0 = y(3)$, find $y(2.25)$ by finite difference method

(ii) Solve : $\nabla^2 u = -4(x + y)$ in the region given $0 \leq x \leq 4$, $0 \leq y \leq 4$ with all boundaries kept at 0 and choosing $\Delta x = \Delta y = 1$. Start with zero vector and do 4 Gauss seidel iterations.

2.(i) Solve $y_{tt} = 4y_{xx}$ subject to the condition $y(0,t) = 0$; $y(2,t) = 0$; $y(x,0) = x(2-x)$; $u_t(x,0) = 0$, Do 4 steps. Find the values upto 2 decimal accuracy.

(ii) Solve $u(x, 0) = 3x$ $0 \leq x \leq 4$, $u(x,4) = x^2$ $0 \leq x \leq 4$, $u(0,y) = 0$ $0 \leq y \leq 4$, $u(4,y) = 12 + y$ $0 \leq y \leq 4$,

3.(i) Solve $y'' = x + y$ with the conditions $y(0) = y(1) = 0$ by finite difference method, taking $h = 0.25$

(ii) $25 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$, $\frac{\partial u}{\partial t}(x,0) = 0$, $u(0,t) = 0$, $u(5,t) = 0$ and $u(x,0) = \begin{cases} 2x, & \text{when } 0 \leq x \leq 2.5 \\ 10 - 2x, & \text{when } 2.5 \leq x \leq 5 \end{cases}$

by the method derived above taking $h=1$ and for one period of vibration, (i.e. up to $t=2$)

4.(i) Solve the Laplace equation $U_{xx} + U_{yy} = 0$ for the square mesh with the boundary values, using Liebmann's method correct to integers

0	100	200	100	0
0	100	200	100	0

5. (i) Solve the elliptic equation $U_{xx} + U_{yy} = 0$ for the following square mesh with boundary values as shown, using Liebman's iteration procedure.

0	11.1	17	19.7	18.6
0	8.7	12.1	12.8	9

6.(i) Solve $U_{xx} + U_{yy} = 0$ over the square mesh of side 4 units, satisfying the following conditions $u(x, 0) = 3x$ $0 \leq x \leq 4$, $u(x,4) = x^2$ $0 \leq x \leq 4$, $u(0,y) = 0$ $0 \leq y \leq 4$, $u(4,y) = 12 + y$ $0 \leq y \leq 4$,

7.(i) Solve the equation $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square mesh with sides $x = 0$, $y = 0$, $x = 3$ and $y = 3$ with $u = 0$ on the boundary and mesh length is 1.

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8(i) Find the value of $u(x, t)$ satisfying the parabolic equation $\frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2}$ and the boundary

conditions $u(0, t) = 0$, $u(8, t) = 0$ and $u(x, 0) = 4x - \frac{x^2}{2}$ at $x = i$, $i = 0, 1, 2, \dots, 7$ and $t = 1/8j$, $j = 0, 1, 2, 3$

$y(2.5)$ and $y(2.75)$

(ii) Solve : $U_{xx} + U_{yy} = 0$ given

A	1	2	B