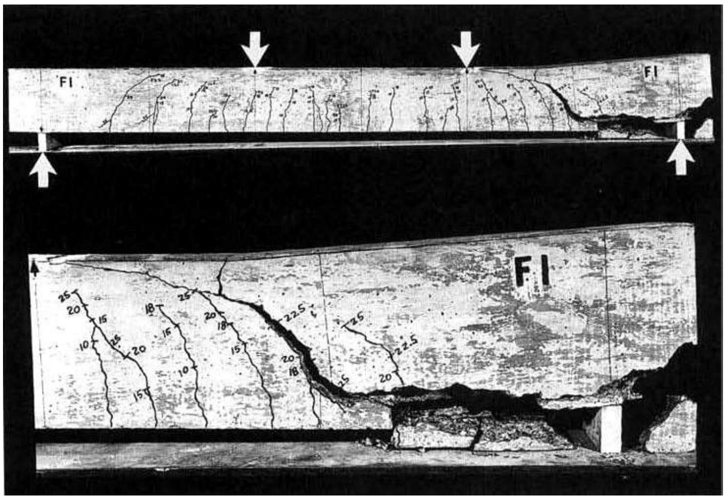
CHAPTER 5 **SHEAR IN BEAMS**

1.1. INTRODUCTION

When a simple beam is loaded, bending moments and shear forces develop along the beam. To carry the loads safely, the beam must be designed for both types of forces. Flexural design is considered first to establish the dimensions of the beam section and the main reinforcement needed, as explained in the previous chapters.

The beam is then designed for shear. If shear reinforcement is not provided, shear failure may occur. Shear failure is characterized by small deflections and lack of ductility, giving little or no warning before failure. On the other hand, flexural failure is characterized by a gradual increase in deflection and cracking, thus giving warning before total failure. This is due to the ACI Code limitation on flexural reinforcement. The design for shear must ensure that shear failure does not occur before flexural failure.



Shear failure of reinforced concrete beam

By the traditional theory of homogeneous, elastic, uncracked beams, we can calculate shear stresses, v, using equation

$$v = \frac{VQ}{Ib}$$

where $V -$ total shear at the section considered,

$Q -$ statical moment about the neutral axis of that portion of cross-section lying between a line through the point in question parallel to the neutral axis and nearest face, upper or lower, of the beam,

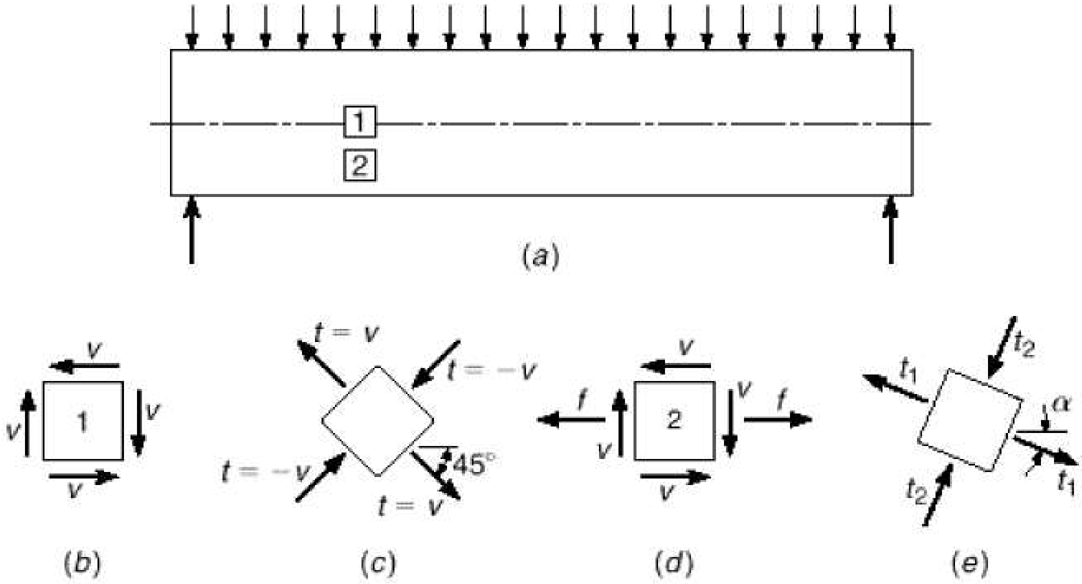
$I -$ moment of inertia of cross-section about the neutral axis,

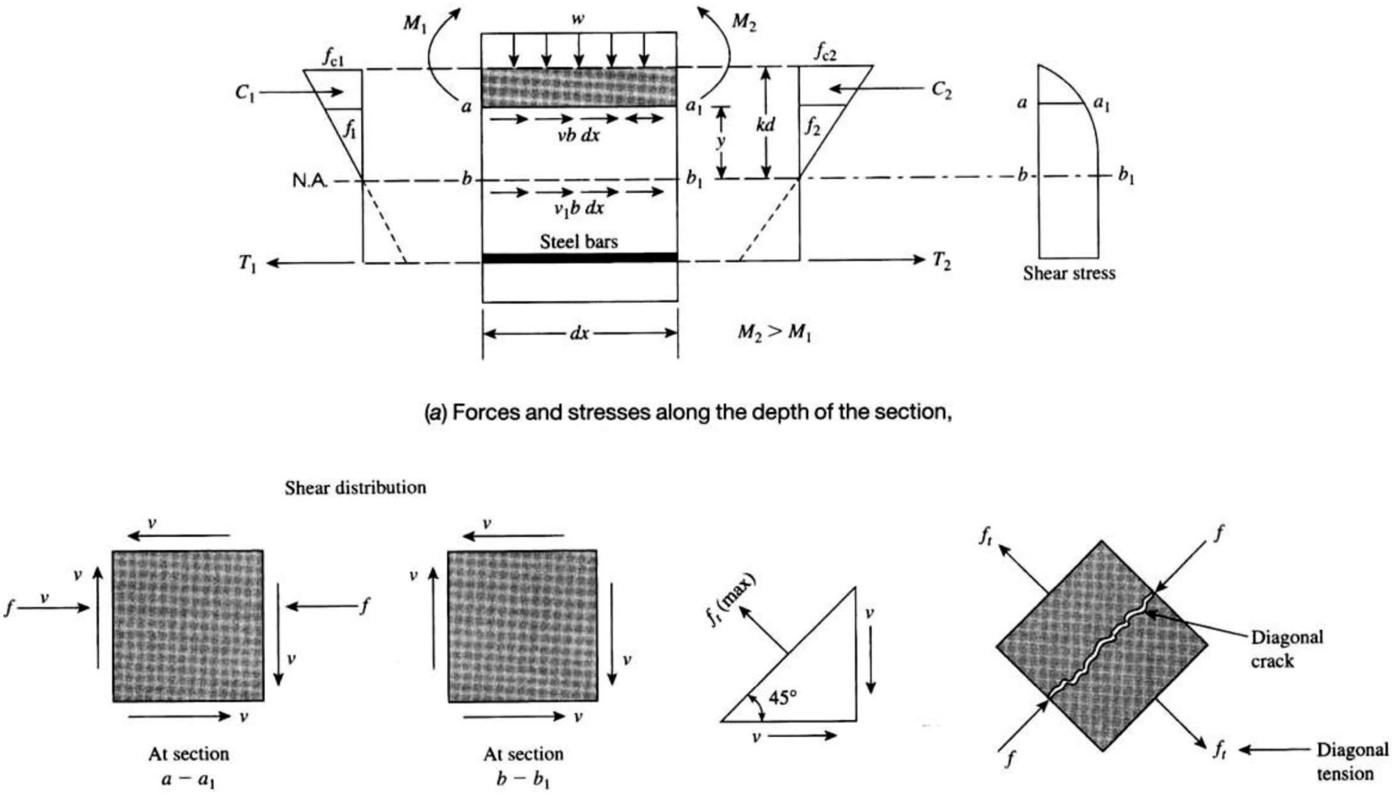
$b -$ width of beam at the given point.

The tensile stresses are equivalent to the principal stresses. Such principal stresses are traditionally called **diagonal tension stresses**. When the diagonal tension stresses reach the tensile strength of concrete, a diagonal crack develops. This brief analysis explains the concept of diagonal tension and diagonal cracking. The actual behavior is more complex, and it is affected by other factors. For the combined action of shear and normal stresses at any point in a beam, the maximum and minimum diagonal tension (principal stresses) f_p are given by the equation

$$f_p = \frac{f}{2} \pm \sqrt{\left(\frac{f}{2}\right)^2 + v^2}$$

f – intensity of normal stress due to bending,
v – shear stress.



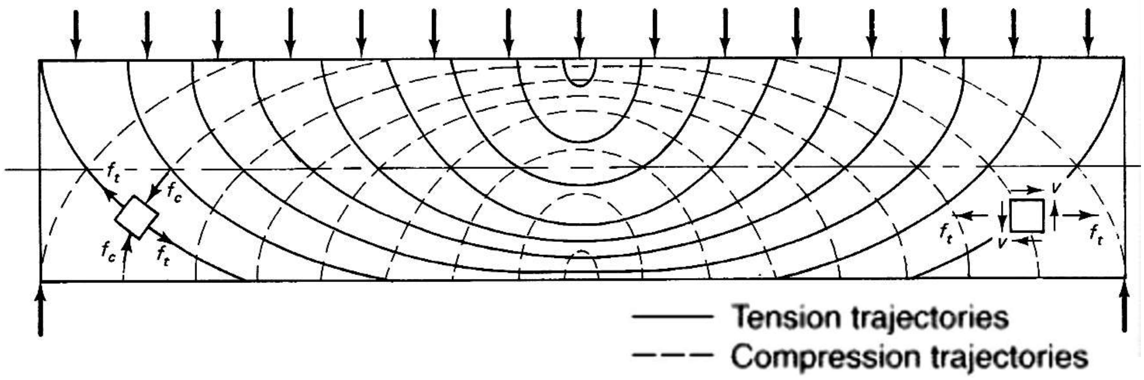


(a) Forces and stresses along the depth of the section,

(b) normal and shear stresses,

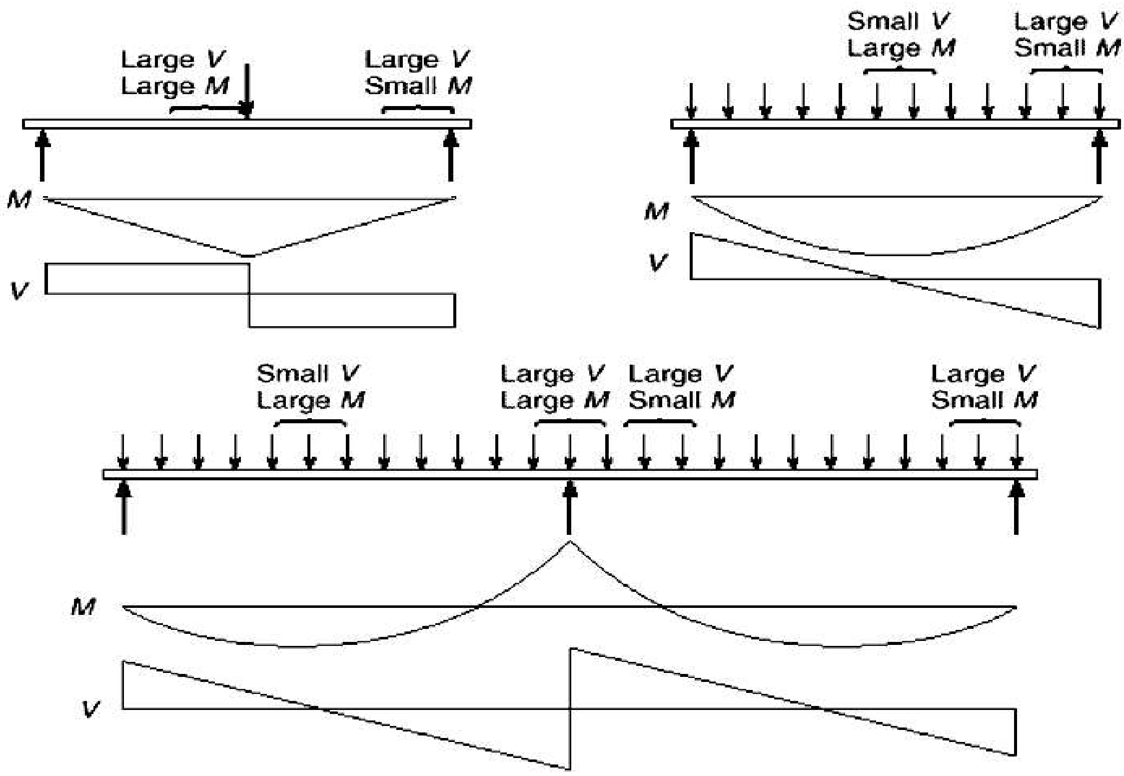
(c) pure shear,

(d) diagonal tension.



Trajectories of principal stresses in a homogeneous isotropic beam.

1.2. CRITICAL SECTIONS FOR SHEAR DESIGN



Typical Locations of critical combinations of shears and moment

In a beam loaded on the top flange and supported on the bottom as shown in the figure below, the closest inclined cracks that can occur adjacent to the supports will extend outward from the supports at roughly 45°. Loads applied to the beam within a distance d from the support in such a beam will be transmitted directly to the support by the compression fan above the 45° cracks and will not affect the stresses in the stirrups crossing the cracks shown. As a result, ACI Code Section 11.1.3.1 states:

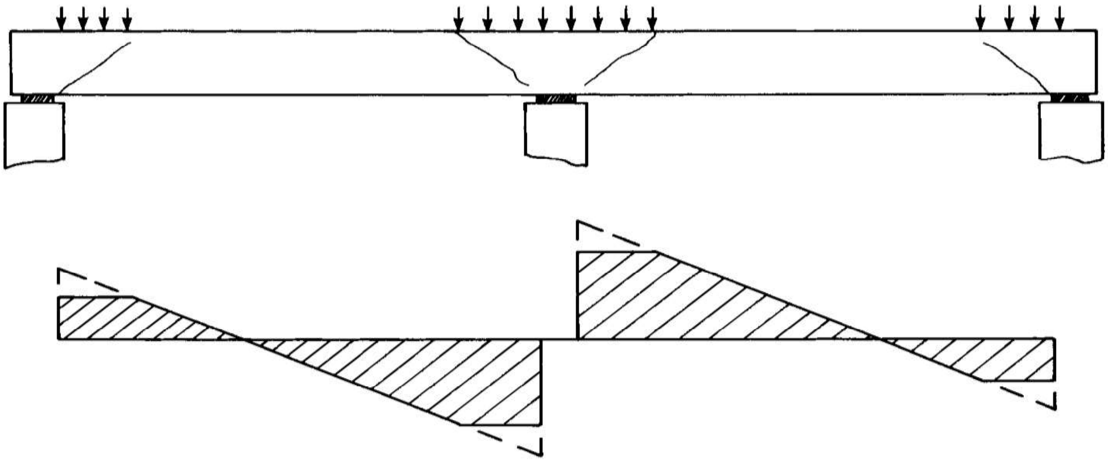
For nonprestressed members, sections located less than a distance d from the face of the support may be designed for the same shear, V_u, as that computed at a distance d.

Reinforced Concrete (1)- 3rd Year, Civil Engineering

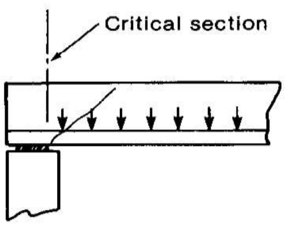
This is permitted only when

1. the support reaction, in the direction of the applied shear, introduces compression into the end regions of a member,
2. the loads are applied at or near the top of the beam, and
3. no concentrated load occurs within d from the face of the support.

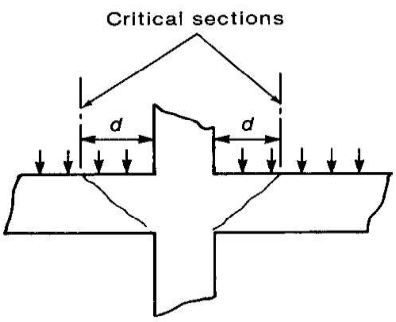
Thus, for the beam shown below, the values of V_u used in design are shown shaded in the shear force diagram.



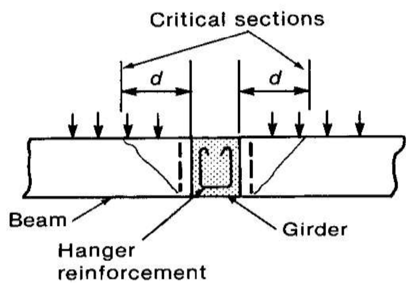
This allowance must be applied carefully because it is not applicable in all cases. There are shows five other typical cases that arise in design. If the beam was loaded on the lower flange, as indicated in Fig. a, the critical section for design would be at the face of the support, because loads applied within d of the support must be transferred across the inclined crack before they reach the support.



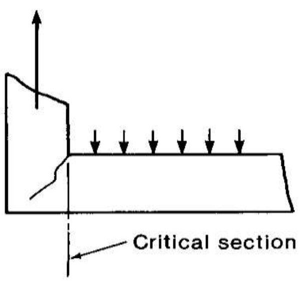
(a) Beam loaded on tension flange.



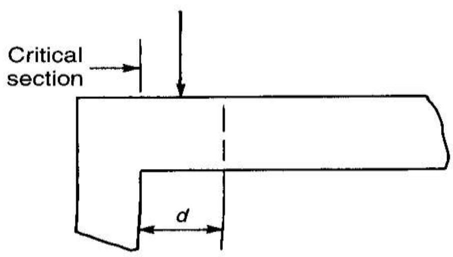
(b) Beam column joint.



(c) Beam supported by shear.



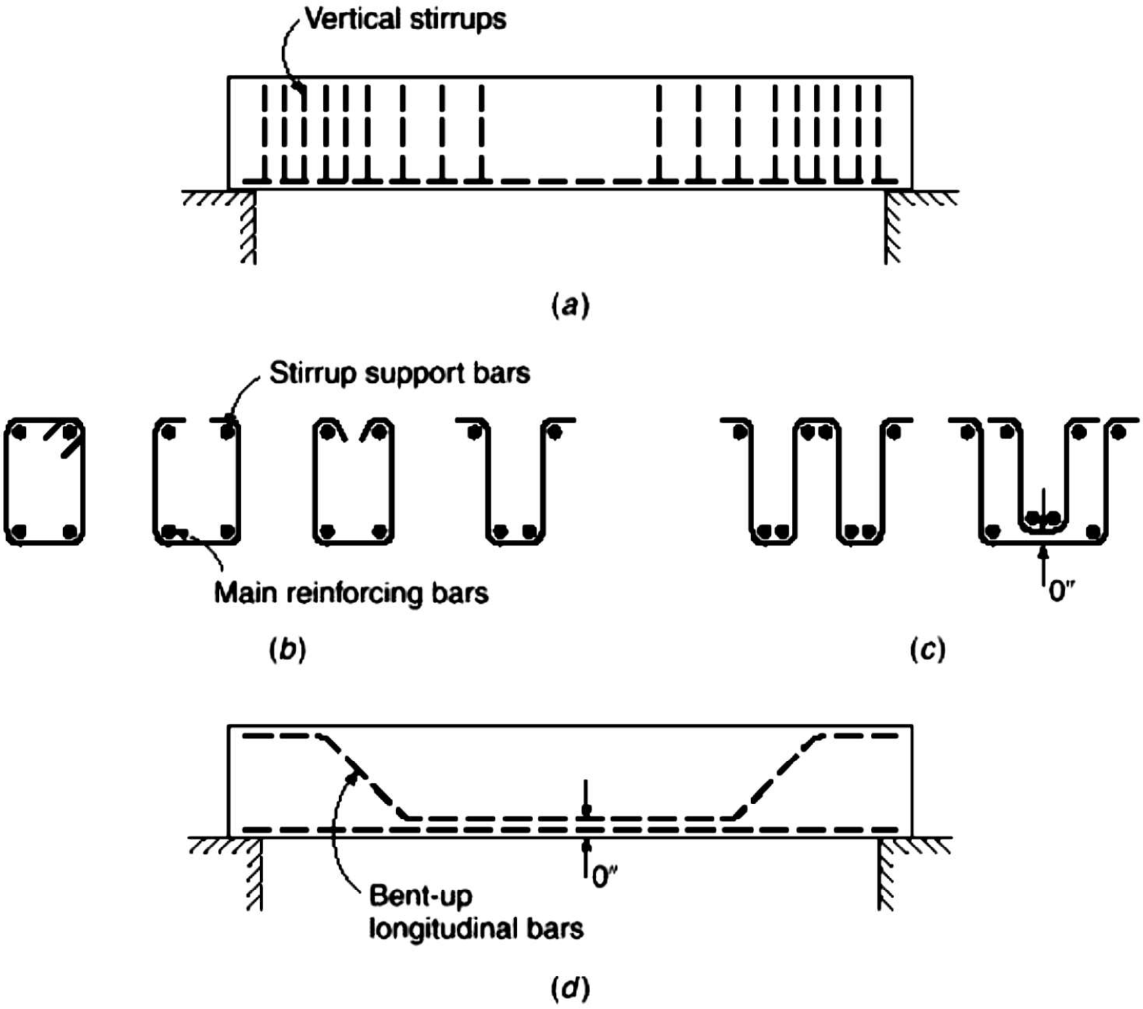
(d) Beam supported by tension force.



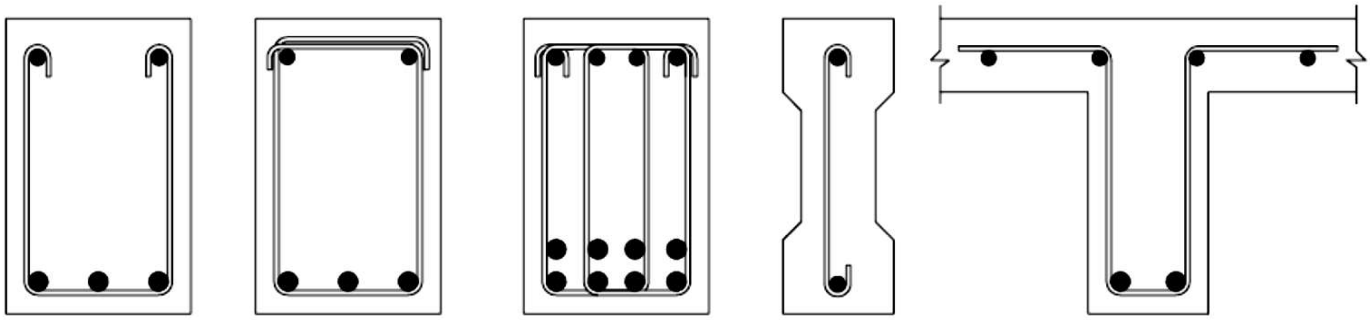
(e) Beam with concentrated load close to support.

A typical beam-to-column joint is shown in Fig. b. Here the critical section for design is d away from the section as shown.
If the beam is supported by a girder of essentially the same depth, as shown in Fig. c, the compression fans that form in the supported beams will tend to push the bottom off the supporting beam. The critical shear design sections in the supported beams normally are taken at the face of the supporting beam. The critical section may be taken at d from the end of the beam if hanger reinforcement is provided to support the reactions from the compression fans.
Generally, if the beam is supported by a tensile force rather than a compressive force, the critical section will be at the face of the support, and the joint must be carefully detailed, because shear cracks will extend into the joint, as shown in Fig. d.
Occasionally, a significant part of the shear at the end of the beam will be caused by a concentrated load acting less than d from the face of the column, as shown in Fig. e. In such a case, the critical section must be taken at the support face.

1.3. TYPES OF WEB REINFORCEMENT



a) Vertical ***Stirrups,***
b) U-shaped bars single stirrups.
c) Multiple-leg stirrups
d) Bent-up longitudinal (inclined) bars



The ACI Code defines the types of shear reinforcement as:

11.4.1.1 — Shear reinforcement consisting of the following shall be permitted:

(a) Stirrups perpendicular to axis of member;

(b) Welded wire reinforcement with wires located perpendicular to axis of member;

(c) Spirals, circular ties, or hoops.

11.4.1.2 — For nonprestressed members, shear reinforcement shall be permitted to also consist of:

(a) Stirrups making an angle of 45 degrees or more with longitudinal tension reinforcement;

(b) Longitudinal reinforcement with bent portion making an angle of 30 degrees or more with the longitudinal tension reinforcement;

(c) Combinations of stirrups and bent longitudinal reinforcement.

1.4. DESIGN PROCEDURE FOR SHEAR

Design of cross section subjected to shear shall be based on:

$$\phi V_n \geq V_u$$

where V_u — the factored shear force at the section,

V_n — the nominal shear strenght,

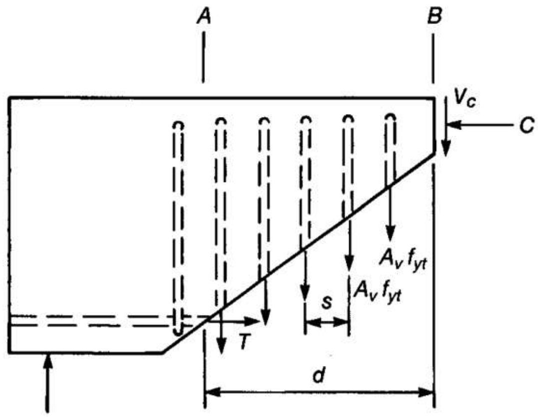
$$V_n = V_c + V_s,$$

V_c — the nominal shear strenght provided by concrete,

V_s — the nominal shear strenght provided by shear reinforcement (stirrups),

The figure shows a free body between the end of a beam and an inclined crack. The horizontal projection of the crack is taken as d, suggesting that the crack is slightly flatter

than 45°. If s is the stirrup spacing, the number of stirrups cut by the crack is d/s. Assuming that all the stirrups yield at failure, the shear resisted by the stirrups is



$$V_s = \frac{A_v f_{yt} d}{s}$$

ACI Code 11.2.1 states, for members subject to shear and flexure only

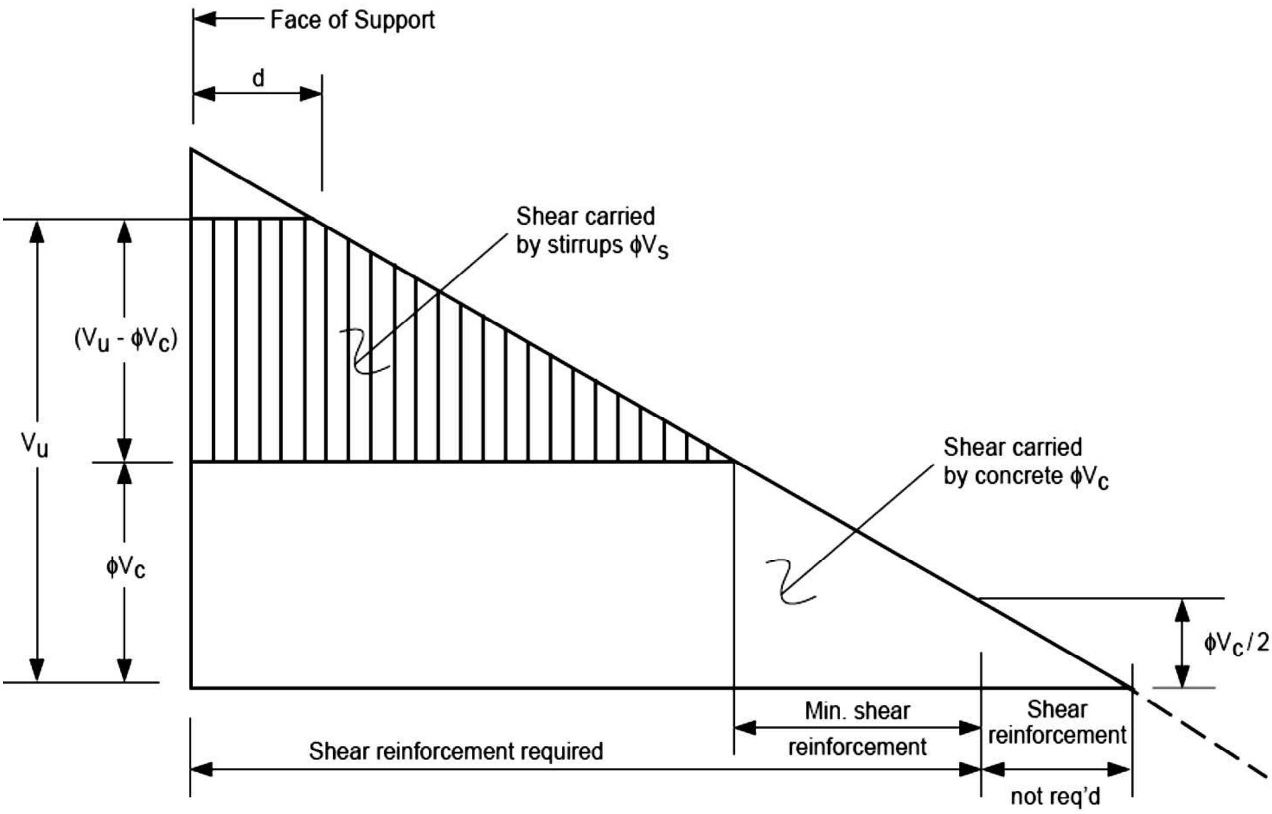
$$V_c = \frac{1}{6}\lambda\sqrt{f_c'}b_w d = 0.17\lambda\sqrt{f_c'}b_w d, \qquad \lambda = 1.0 \ \ for\ Normal - weight\ concrete$$

V_c shall be permitted to be computed by the more detailed calculation

$$V_c = \left(0.16\lambda\sqrt{f_c'} + 17\rho_w \frac{V_u d}{M_u}\right) b_w d \le 0.29\sqrt{f_c'}b_w d, \qquad where \qquad \frac{V_u d}{M_u} \le 1$$

To simplify the calculations the formula $V_c = 0.17\lambda\sqrt{f_c'}b_w d$ will be used.

Shear conditions and cases (Items):



Check for dimensions:

The ACI Code, 11.4.7.9, states that V_s shall not be taken greater than $0.66\sqrt{f_c'}b_w d$.

$$So, \quad if \quad V_s > V_{s,max} - The\ section\ must\ be\ enlarged\ (Dimenstions\ are\ not\ enough)$$

$$where \qquad V_s = V_n - V_c = \frac{V_u}{\phi} - V_c, \qquad V_{s,max} = \frac{2}{3}\sqrt{f_c'}b_w d$$

Case I:

$$V_u \le \frac{1}{2}\phi V_c \quad - No\ shear\ reinforcement\ is\ required$$

Case II:

$$\frac{1}{2}\phi V_c < V_u \le \phi V_c \quad - Minimum\ shear\ reinforcement\ is\ required\ \left(A_{v,min}\right)\ except:$$

- footings and solid slabs,
- Hollow-core units with total untopped depth not greater than 315 mm and hollow-core units where V_u is not greater than $0.5\phi V_{cw}$;
- Concrete joist construction;
- Beams with h not greater than $250\ mm$;
- Beam integral with slabs with h not greater than $600\ mm$ and not greater than the larger of 2.5 times thickness of flange, and 0.5 times width of web;
- Beams constructed of steel fiber-reinforced, normalweight concrete with f_c' not exceeding 40 MPa, h not greater than 600 mm, and V_u not greater than $0.17\sqrt{f_c'}b_w d$.

For these cases no shear reinforcement is required unless $V_u > \phi V_c$.

Minimum shear reinforcement, $A_{v,min}$

$$A_{v,min} = \frac{1}{16}\sqrt{f_c'}\frac{b_w s}{f_{yt}} = 0.062\sqrt{f_c'}\frac{b_w s}{f_{yt}} \ge \frac{1}{3}\frac{b_w s}{f_{yt}} = 0.35\frac{b_w s}{f_{yt}},$$

$$or\ in\ the\ form \qquad \left(\frac{A_{v,min}}{s}\right) \ge \frac{1}{3}\frac{b_w}{f_{yt}} \ge \frac{1}{16}\sqrt{f_c'}\frac{b_w}{f_{yt}} ,$$

$$Here \qquad s_{max} \le \frac{d}{2} \qquad or \qquad s_{max} \le 600\ mm$$

where $\quad s -$ step of stirrups (spacing between stirrups),

$\quad f_{yt} -$ yield stress of stirrups

Case III:

$$\phi V_c < V_u \le \phi\left(V_c + V_{s,min}\right)$$

$$\frac{A_{v,min}}{s} = \frac{V_{s,min}}{f_{yt}d} \Longrightarrow \qquad V_{s,min} = \left(\frac{A_{v,min}}{s}\right)f_{yt}d$$