F/B - 7

الفرقة الأولى

مدنى

فلويد

د/[redacted] Fluid مبنى - الاولى 7 1 بلحة

7 FUNDAMENTALS OF FLUID FLOW

DEFINITIONS

1. Steady and Unsteady Flow

- **Steady Flow:-**
  Flow properties (velocity) don't change with time

  $\rightarrow \frac{dv}{dt}=0$

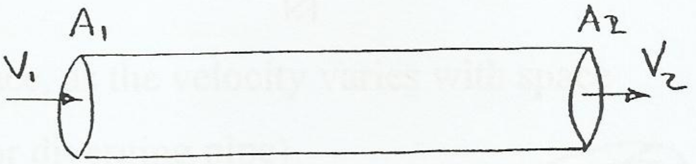
- **Un-steady Flow:-**
  Flow properties change with time

  $\rightarrow \frac{dv}{dt}\neq 0$

2. Uniform and Non-uniform Flow:

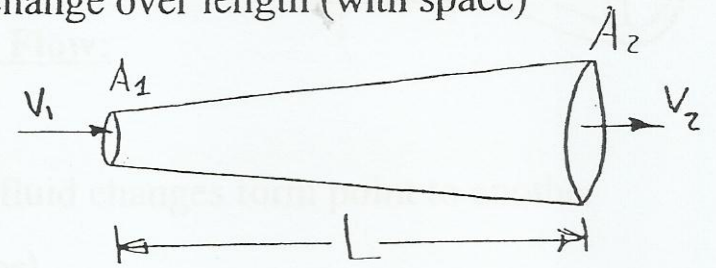
- **Uniform Flow:-**
  Flow properties (velocity, cross-section) are constant over a given length (don't change with space)

  $\rightarrow \frac{dv}{dL}=0 \quad \frac{dA}{dL}=0$



- **Non- uniform Flow:**
  Flow velocity and cross-section change over length (with space)

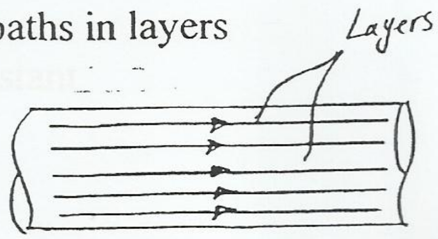
  $\rightarrow \frac{dv}{dL}\neq 0 \quad \frac{dA}{dL}\neq 0$

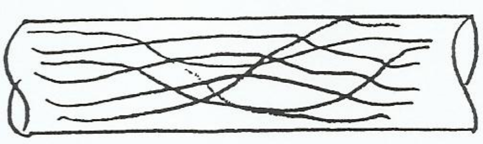


3. Laminar and Turbulent Flow:

- **Laminar Flow:**
  When fluid particles move along straight parallel paths in layers

  $\rightarrow R_N \leq 2000 (R_N = \text{Reynold's number})$





- **Turbulent Flow:** (In Most of cases)

  When fluid particles move in a random motion at $R_N \geq 4000$

  When $2000 < R_N < 4000 \rightarrow$ the flow is <u>Transitional</u>

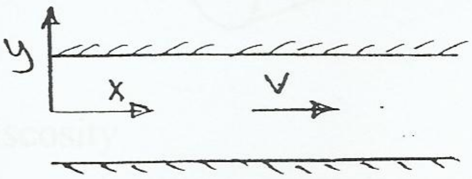
4. **<u>Rotational and Irrortational Flow:</u>**

   If the fluid particles have rotation about a certain axis, then the flow is "Rotational" otherwise it's "Irrotational"

5. **<u>One, Two and Three dimensional flow:</u>**

- **One dimensional flow:**

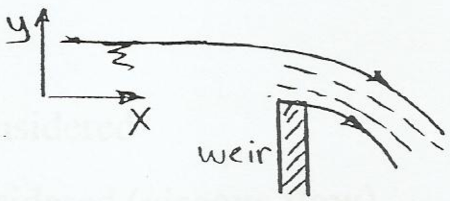
  When the change of flow properties (velocity & pressure) perpendicular to a streamline is **negligible**, then the flow is one dimensional (e.g. flow in pipes). $v = f(x, t)$



- **Two dimensional flow:**

  It is defined by streamline in a single plane (e.g. Flow over weirs)

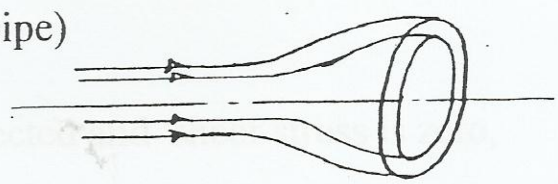
  $v = f(x, y, t)$



- **Three dimensional flow:**

  It is defined by stream-tube in space, as the velocity varies with space (e.g. Flow through a converging or diverging pipe)

  $v = f(x, y, z, t)$



6. **<u>Compressible and Incompressible Flow:</u>**

- **Compressible flow:**

  It occurs when the density of the fluid changes form point to another (e.g. flow of gases through nozzles)

- **Incompressible flow:**

  It occurs when the density of the fluid remains constant (all liquids are considered incompressible)

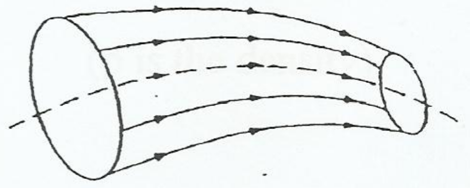
7. **Streamlines and stream-tube:**

- **Streamlines:**

  They are the imaginary curves illustrating the direction of flow for fluid particles. The tangent at any point gives the velocity direction.

- **Stream-Tube:**

  It is the fluid mass bounded by a group of streamlines which are confining the flow



8. **Ideal and Real Fluids:**

- **Ideal Fluid:**
  - The fluid is assumed to have no viscosity
  - No shear resistance is considered (**inviscid flow**)
  - Ideal fluid does not actually exists (as all fluids has viscosity)
- **Real Fluid:**
  - The viscosity of the fluid is considered
  - Shear resistance to flow is considered (**viscous flow**)
  - Due to shear resistance, head losses occurs

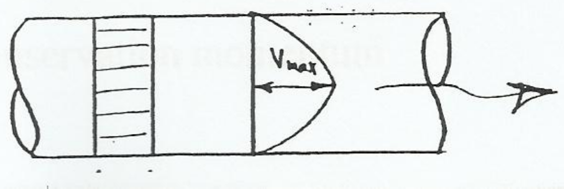
9. **Viscous and Inviscid flows:**

- When the viscosity of flowing fluid is neglected and shear stress is zero, the flow is invisced (No losses exists)
- When the viscosity and shear stress are considered, the flow is viscous (head losses exists)

10. **Mean velocity:**

It is the average velocity of the flow passing through a given section.

$$v_{mean} = \frac{\text{Discharge}}{\text{Area}} = \frac{Q}{A}$$



11. Volume, Mass and Weight flow rates:

- **Volume flow rate (Discharge): (Q)**

  It's the rate of volume of liquid passing through a given section.

  $$\text{Discharge} = \frac{\text{volume}}{\text{Time}}$$

  & $Q = A \times v$ → **continuity Equation**

- **Mass flow rate: (Q_m)**

  It's the rate of liquid mass passing through a given section.

  $$Q_{mass} = \frac{\text{Mass}}{\text{Time}} = \frac{\rho v}{T} \qquad \therefore Q_{mass} = \rho \times Q \qquad (\rho \text{ is the density})$$

- **Weight flow rate: (Q_w)**

  It's the rate of liquid weight passing a given section>

  $$Q_{weight} = \frac{\text{Weight}}{\text{Time}} = \frac{\gamma \times v}{T} \qquad \therefore Q_{weight} = \gamma \times Q \qquad (\gamma \text{ is the specific weight})$$

Example (1):

Volume of water is collected in a tank of a volume $2m^3$ in 30 min.
Calculate: the discharge and the mass and weight rates of flow

* Discharge (volume rate of flow) $= \frac{V}{T} = \frac{2}{30 \times 60} = 1.11 \times 10^{-3}\ m^3/s$

* Mass flow rate $= Q * \rho = 1.11 \times 10^{-3} * 1000 = 1.11\ Kg/s$

* Weight flow rate $= Q * \gamma = 1.11 \times 10^{-3} * 9810 = 10.9\ N/s$

12. The main dynamic equations for fluid flow

To deal with any problem of fluid flow, three main equations are considered:

- Continuity Equation → for the conservation of mass.
- Energy Equation → for the conservation energy
- Momentum Equation → for the conservation momentum

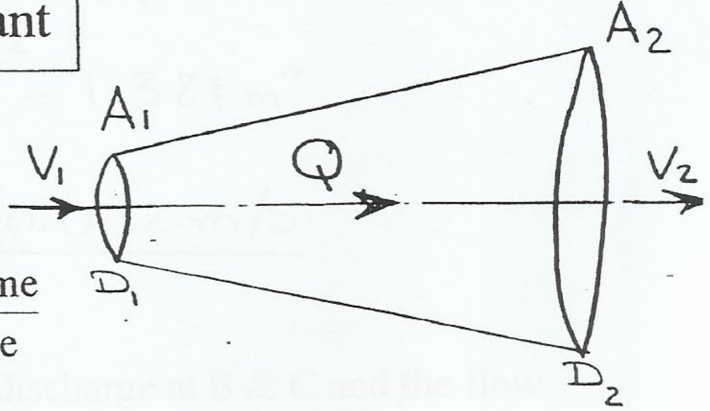
CONTINUITY EQUATION

(For steady, one dimensional incompressible flow)

For a given length of flow between two different sections, if no fluid is added or removed then the flow mass passing the two sections remains constant.

$\therefore$ Continuity Equation between any 2 sections:

$$\boxed{A_1 v_1 = A_2 v_2 = Q = \text{constant}}$$



Where

Q = Discharge = volume flow rate

- Discharge = Area × velocity $= \dfrac{\text{volume}}{\text{Time}}$

- Dimensions $\rightarrow Q = L^2 \times L/T = \dfrac{L^3}{T}$

- Units $\rightarrow Q = 30 \begin{cases} m^3/\text{sec} \\ cm^3/\text{sec} \\ ft^3/\text{sec (cfs)} \end{cases}$

Note:

since $A_1 < A_2$

$\therefore \quad v_1 > v_2$

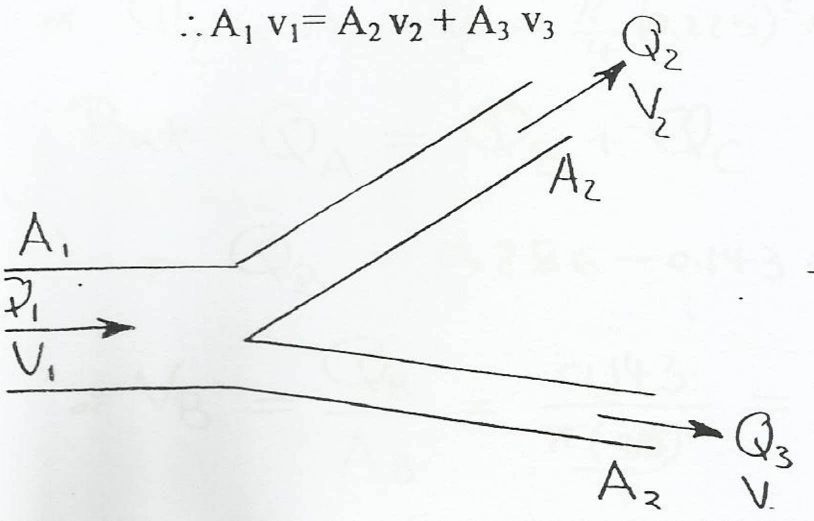
$\rightarrow v_1 = \dfrac{A_2}{A_1} v_2 = \left(\dfrac{D_2}{D_1}\right)^2 \times v_2$

Note

For any problem: $\mathbf{Q_{input} = Q_{output}}$

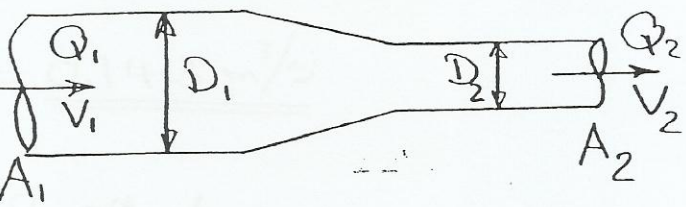
$Q_1 = Q_2 + Q_3$

$\therefore A_1 v_1 = A_2 v_2 + A_3 v_3$



$Q_1 = Q_2$

$\therefore A_1 v_1 = A_2 v_2$



Example (2):

A pipe with a diameter 1300mm transmits 2 million tons of crude oil (S.G = 0.92) per year. Calculate the flow rate and the velocity of flow.

Given... $Q_w = 2\times10^6$ ton/year (weight rate of flow) $*(g)$ 1 ton = 1000 Kg

$\rightarrow Q = Q_w/\gamma = \dfrac{1}{0.92\times9810} * \dfrac{(2\times10^6)(9.81\times1000)}{365\times24\times60\times60}$

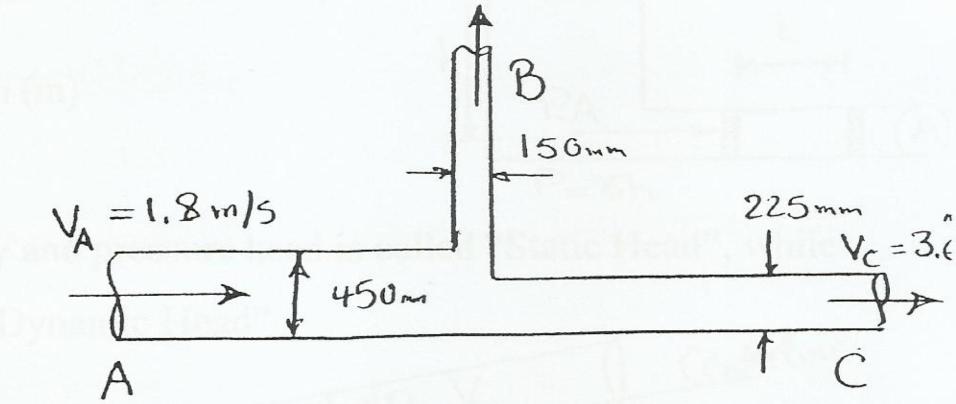
$\rightarrow \underline{Q = 0.069\ m^3/s} = V * A$

Area of pipe $= \dfrac{\pi D^2}{4} = \dfrac{\pi(1.3)^2}{4} = 1.327\ m^2$

$\therefore V = \dfrac{Q}{A} = \dfrac{0.069}{1.327} = \underline{0.052\ m/s}$

Example (3):

For the shown pipes junction, calculate the discharge at B & C and the flow velocity at B



$* Q_A = A_A * V_A$

$= \frac{\pi}{4}(0.45)^2 * 1.8 = 0.286\ m^3/s$

$* Q_C = A_C * V_C = \frac{\pi}{4}(0.225)^2 * 3.6 = \underline{\underline{0.143\ m^3/s}}$

But $Q_A = Q_B + Q_C$

$\rightarrow Q_B = 0.286 - 0.143 = \underline{\underline{0.143\ m^3/s}}$

$\therefore V_B = \dfrac{Q_B}{A_B} = \dfrac{0.143}{\frac{\pi(0.15)^2}{4}} = \underline{\underline{8.09\ m/s}}$

FLUID ENERGY

For any fluid element, there are three quantities of energy or heads.

1. **<u>Potential energy ≡ potential (elevation) head</u>**

$$\text{Potential head} = \frac{\text{potential energy}}{\text{unit weight}} = \frac{m\,g \times Z}{m\,g} = Z$$

$\therefore$ Elevation head = Height above datum (Z)

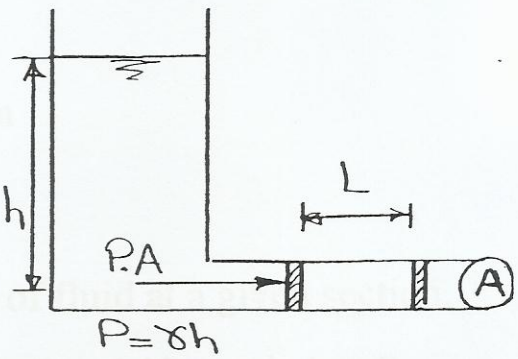
2. **<u>Kinetic energy ≡ Velocity head.</u>**

$$\text{Velocity head} = \frac{\text{kinetic energy}}{\text{unit weight}} = \frac{mv^2/2}{mg} = \frac{v^2}{2g} \quad \therefore \text{ Velocity head} = \frac{v^2}{2g}\,(\text{m})$$

3. **<u>Pressure energy ≡ pressure head</u>**

$$\text{Pressure head} = \frac{\text{pressure energy}}{\text{unit weight}} = \frac{P \times A \times L}{\gamma A L} = \frac{P}{\gamma} = h$$

$$\therefore \text{Pressure head} = \frac{P}{\gamma} = h\,(\text{m})$$

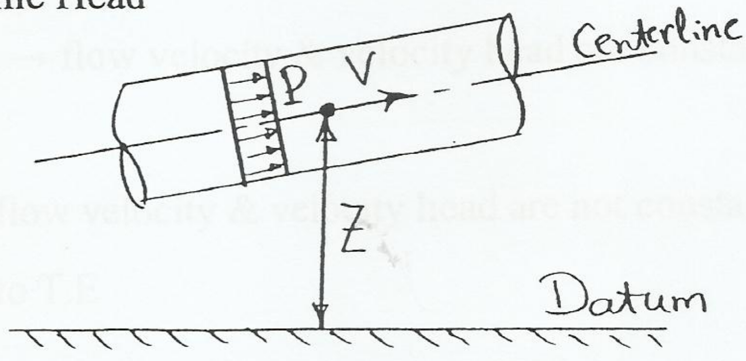


<u>Note:</u>

The sum of elevation energy and pressure head is called "Static Head", while the velocity head is called "Dynamic Head"

$$\text{Static head} = Z + \frac{P}{\gamma}$$

$$\text{Dynamic head} = \frac{v^2}{2g}$$



<u>At any section:</u>

$$\text{Total head} = \overbrace{\text{Elevation head} + \text{Pressure head}}^{\text{Static Head}} + \overbrace{\text{velocity head}}^{\text{Dynamic Head}}$$

$$\boxed{H = Z + \frac{P}{\gamma} + \frac{v^2}{2g}}$$

Note: all heads are in (m) or (ft)

T.E.L & H.G.L

➢ Total Energy Line (T.E.L):

It's a line that represents the **total head** of the fluid at a given section.

a) **For ideal fluid:-**

$$H = Z + \frac{v^2}{2g} + \frac{P}{\gamma} = \text{constant}$$

$\therefore$ T.E.L is always horizontal (T.E.L // Datum)

b) **For real fluid:-**

$$H = Z + \frac{v^2}{2g} + \frac{P}{\gamma} \neq \text{constant} \qquad (H_1 - h_{loss} = H_2)$$

$\therefore$ T.E.L is always inclined and **not parallel** to datum

➢ Hydraulic Gradient Line (H.G.L):

It's a line which represents the **static head** ($P/\gamma + Z$) of fluid at a given section.

$\therefore$ The vertical distance bet T.E.L & H.G.L = velocity head (dynamic head)

Note

- If the area of flow is constant → flow velocity & velocity head are constant

  $\therefore$ H.G.L // T.E.L

- If the area is not constant → flow velocity & velocity head are not constant

  $\therefore$ H.G.L is not parallel to T.E

Note

- The H.G.L is considered as an imaginary free surface as it passes through the points of zero pressure
- From Datum to pipe center line = Z
- From C.L to H.G.L = P/γ
- from H.G.L to T.E.L = $\frac{v^2}{2g}$

T.E.L & H.G.L for ideal fluid:

For ideal fluid, the flow is frictionless (No losses is considered)

$$\therefore H_1 = H_2 \quad \rightarrow \quad \frac{P_1}{\gamma} + Z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\gamma} + Z_2 + \frac{v_2^2}{2g}$$

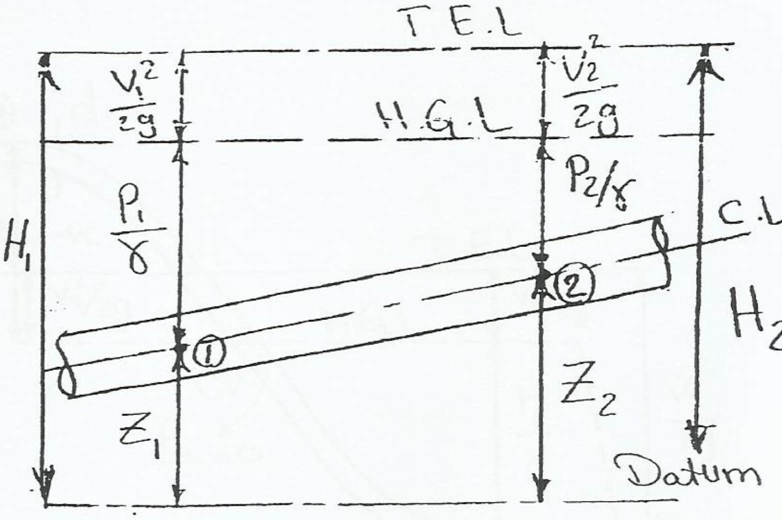
Case (1):

- No losses → T.E.L // Datum
- Cross section area = constant

$A_1 = A_2 \rightarrow v_1 = v_2$

$\therefore \frac{v_1^2}{2g} = \frac{v_2^2}{2g}$

→ T.E.L//H.G.L



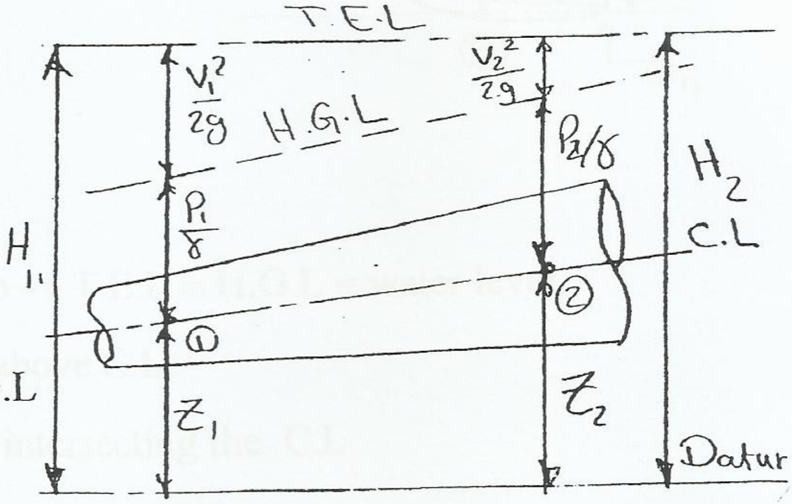
Case (2):

- No losses → T.E.L // Datum
- Cross section area ≠ constant

$A_1 < A_2$

$\rightarrow v_1 > v_2 \quad \therefore \frac{v_1^2}{2g} > \frac{v_2^2}{2g}$

→ T.E.L not parallel to H.G.L



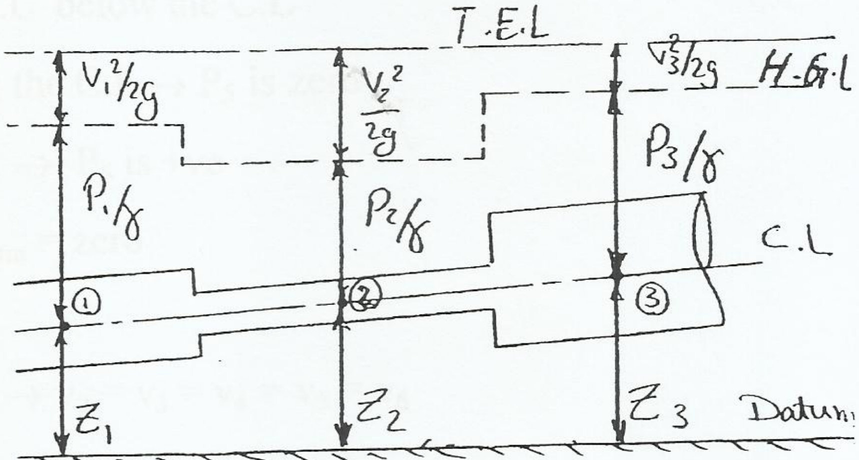
Case (3):

- No losses → T.E.L // Datum
- Cross section area ≠ constant

$A_3 > A_1 > A_2$

$\rightarrow v_3 < v_1 < v_2$

$\therefore \frac{v_3^2}{2g} < \frac{v_1^2}{2g} < \frac{v_2^2}{2g}$

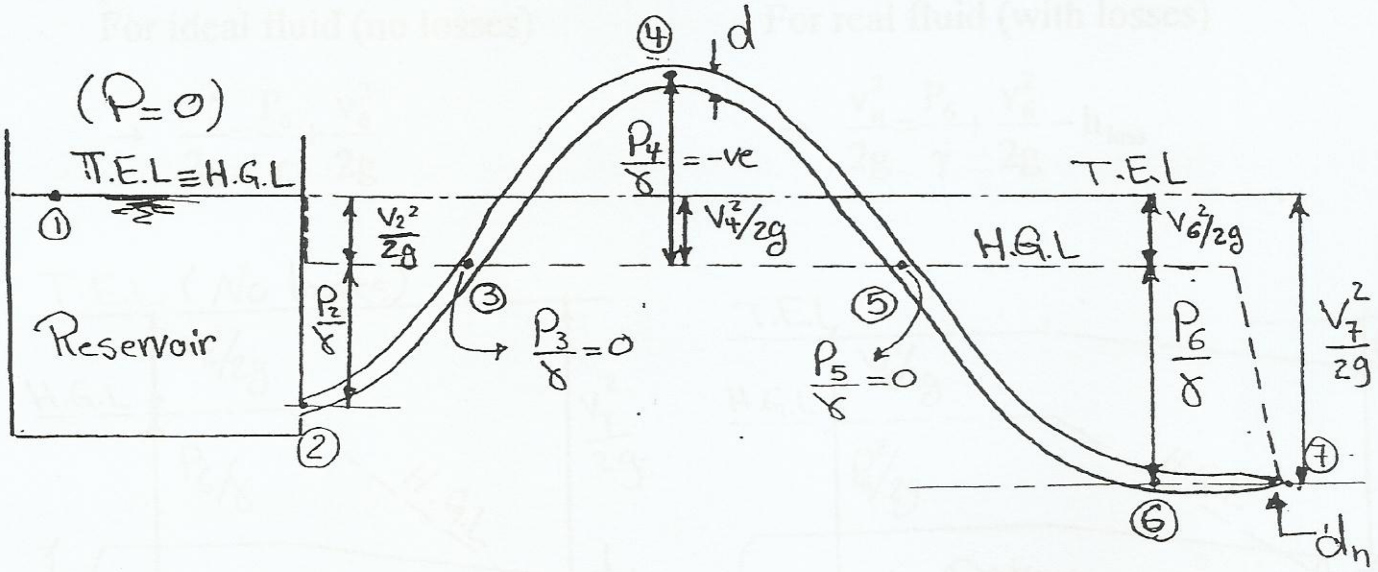


Note

When the area of cross section is decreased suddenly, the velocity increases suddenly and the H.G.L goes down or up suddenly!!

Relation bet. H.G.L & pipe center liner:

1. If the pressure is +ve → H.G.L above C.L
2. If the pressure is -ve → H.G.L below C.L
3. If the pressure is zero → H.G.L intersects C.L



- Point (1) [At reservoir]

  $P_1 = P_{atm} =$ zero $\qquad v_1 =$ Zero → T.E.L ≡ H.G.L ≡ water level.

- Point (2) P_2 is +ve → H.G.L above C.L
- Point (3) P_3 is zero → H.G.L intersecting the C.L
- Point (4) P_4 is –ve → H.G.L below the C.L
- Point (5) H.G.L intersecting the C.L → P_5 is zero
- Point (6) H.G.L. above C.L → P_6 is +ve
- Point (7) [At nozzle] $P_7 = P_{atm} =$ zero

Note

Pipe diameter (d) is constant → $v_2 = v_3 = v_4 = v_5 = v_6$

$\therefore \frac{v^2}{2g} =$ const → H.G.L // T.E.L along the whole pipe except the nozzle

➤ Nozzle

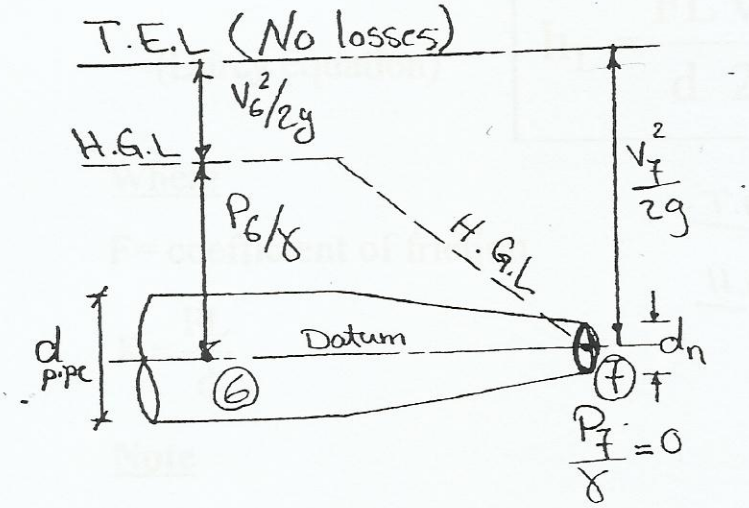
- $d_n < d_{pipe}$ $\rightarrow$ $v_n > v_{pipe}$ $\therefore \frac{v_n^2}{2g} = \frac{v_7^2}{2g} > \frac{v_6^2}{2g}$
- Through nozzle, diameter changes $\rightarrow v \neq$ cont $\therefore$ H.G.L not // T.E.L
- At nozzle $P = P_{atm} =$ zero $\rightarrow$ H.G.L intersects the center line!!

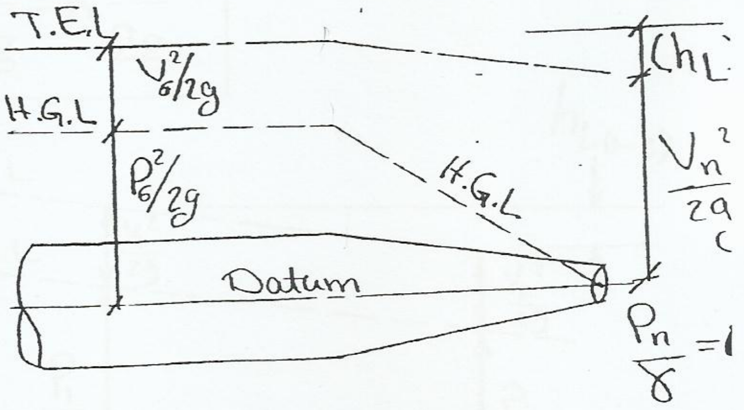
For ideal fluid (no losses)

$$\rightarrow \frac{v_n^2}{2g} = \frac{P_6}{\gamma} + \frac{v_6^2}{2g}$$

For real fluid (with losses)

$$\rightarrow \frac{v_n^2}{2g} = \frac{P_6}{\gamma} + \frac{v_6^2}{2g} - h_{loss}$$

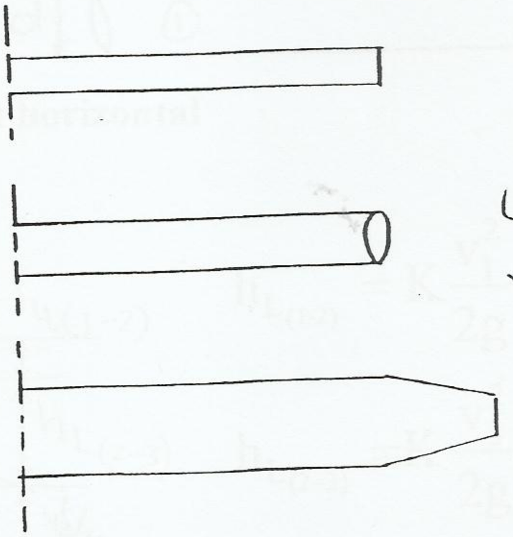




Note

End of the pipe:

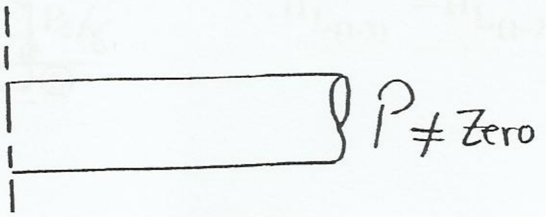
At the end of the pipe P = zero



خد بالك أوي
من الحوار ده!!

This is not the end of the pipe

$\therefore P \neq$ zero

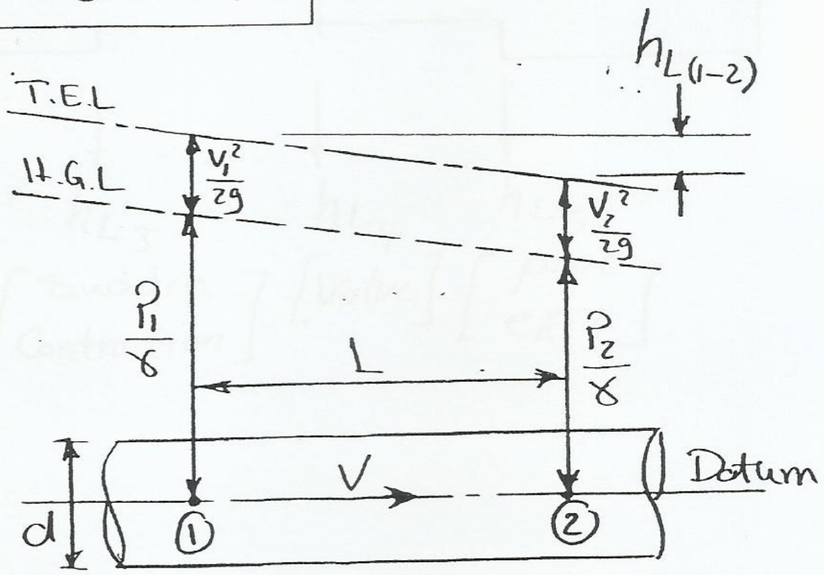


LOSSES IN PIPE FLOW

a) Main losses:

- The main losses in pipes are due to friction of liquid with the walls of the pipe (so it is called **friction losses**)
- Due to friction, the total head decreases with flow
- The frictional head losses is relative to the flow velocity
- To calculate the head losses between two points in a pipe, use this equation

(Darcy equation) $$\boxed{h_L = \frac{FL}{d}\frac{v^2}{2g} = K\frac{v^2}{2g}}$$



Where

F= coefficient of friction

$K = \frac{FL}{d}$

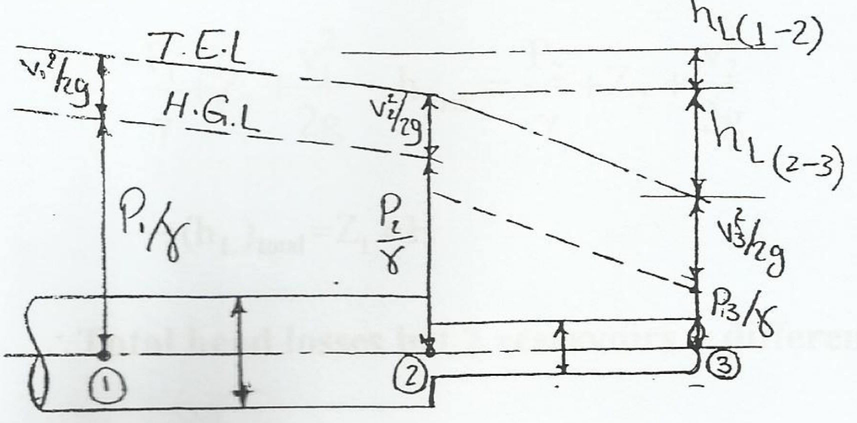
Note

$d_1 = d_2$

$\therefore \frac{v_1^2}{2g} = \frac{v_2^2}{2g}$

$\therefore$ H.G.L // T.E.L **But T.E.L is not horizontal**

Example:



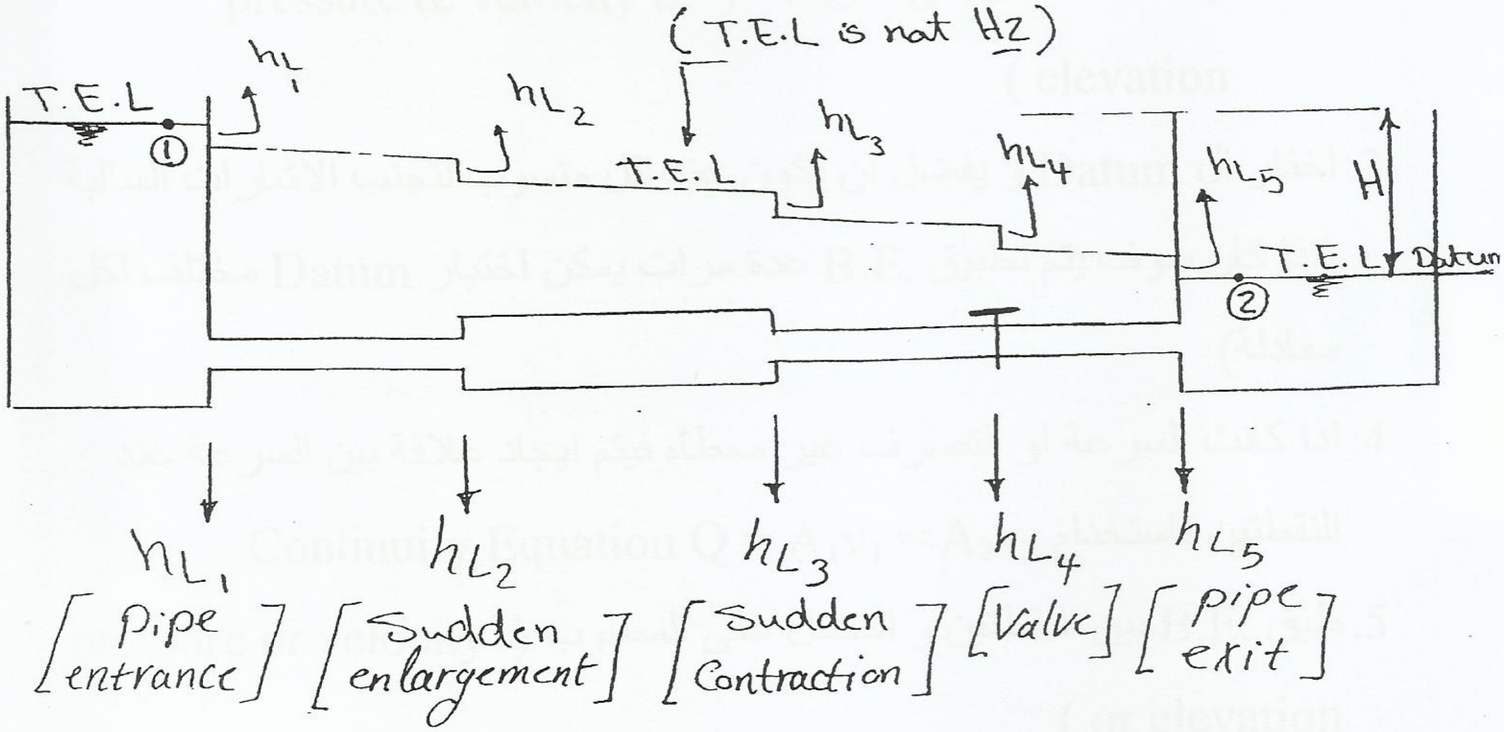
$$h_{L(1\text{-}2)} = K\frac{v_1^2}{2g}$$

$$h_{L(2-3)} = K\frac{v_3^2}{2g}$$

$$\therefore h_{L(1\text{-}3)} = h_{L(1-2)} + h_{L(2-3)}$$

b) Secondary losses:

- It is all the types of losses except frictional losses
- The minor (secondary) losses are due to pipe fittings and valves
- Each type of minor losses has its equation هنذرسها بعدين بالتفصيل الممل



The total head losses from (1) to (2)

$$(h_{L_{total}}) = \underset{(1)-(2)}{h_f} + \sum h_{L_{minor}} = hf + \left[h_{L_1} + h_{L_2} + h_{L_3} + h_{L_{41}} + h_{L_5}\right]$$

Applying B.E bet (1), (2) {datum at (2)}

$$\frac{P_1}{\gamma} + Z_1 + \frac{v_1^2}{2g} - h_{loss} = \frac{P_2}{\gamma} + Z_2 + \frac{v_2^2}{2g}$$

$$\rightarrow (h_L)_{total} = Z_1 = H$$

∴ Total head losses bet 2 reservoirs = difference in water level……!!

Steps to solve Bernoulli's problems

1. ارسم المسألة بوضوح
2. حدد النقطتين التين سوف يتم تطبيق Bernoulli بينهما في (نفس السائل)
   - نقطة عند الشئ المطلوب تحديده (pressure or velocity)
   - نقطة عندها كل شئ معروف (pressure & velocity & elevation)
3. اختار ال Datum و يفضل ان يكون عند اقل منسوب لتجنب الاشارات السالبة (اذا كان سوف يتم تطبيق .B.E عدة مرات يمكن اختيار Datum مختلف لكل معادلة)
4. اذا كانت السرعة او التصرف غير معطاه فيتم ايجاد علاقة بين السرعة عند النقطتين باستخدام Continuity Equation $Q = A_1 v_1 = A_2 v_2$
5. طبق .B.E بين النقطتين و احصل على المطلوب (pressure or velocity or elevation)
6. يتم طرح ال losses اذا كانت معطاه (سوف نحسبها نحن بعد ذلك)
7. اذا كان مطلوب يتم رسم T.E.L & H.G.L
8. First, draw T.E.L

   It has two cases: Horizontal (if the fluid is ideal = no losses)

   Inclined (if there is losses)
9. Then draw H.G.L from T.E.L

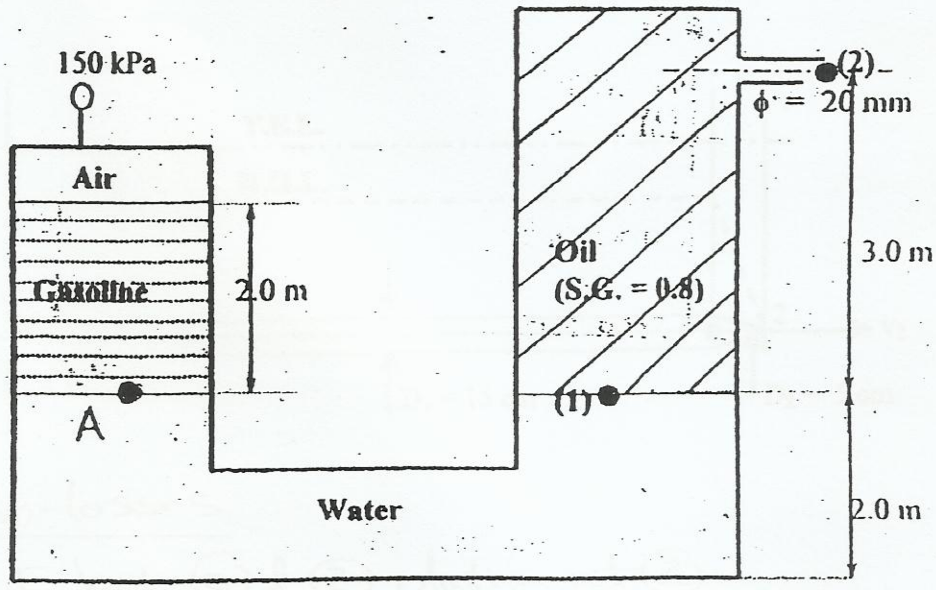
   It has two cases: H.G.L // T.E.L if the diameter (velocity) is constant

   H.G.L not // T.E.L if the diameter (velocity) is variable

Ex. 4

A large tank contains compressed air, gasoline (S.G. = 0.68), light oil (S.G = 0.80) and water as shown in figure. The pressure P of the air is 150 KPa gage. If friction is neglected, what is the mass flow of oil from a 20-mm diameter jet?

Solution:



Since $P_1 = P_A = P_{air} + h \times \gamma_{gas}$

$\therefore P_1 = 150\times10^3 + 2*(0.68\times9810) = 163342$ Pa

Apply B.E bet ①, ② datum at ①

$$0 + \frac{P_1}{\gamma} + 0 = 3 + 0 + \frac{V_2^2}{2g}$$

$$\frac{V_2^2}{2g} = \frac{163342}{0.8\times9810} - 3 \Rightarrow V_2 = 18.69\ m/s$$

$$\therefore Q = A_2 V_2 = \frac{\pi(0.02)^2}{4} * 18.69 = 5.87\times10^{-3}\ m^3/s$$

$\therefore$ Mass flow rate $= Q * \rho$

$$= 5.87\times10^{-3} * (0.8\times1000)$$

$$= \boxed{4.7\ kg/s}$$

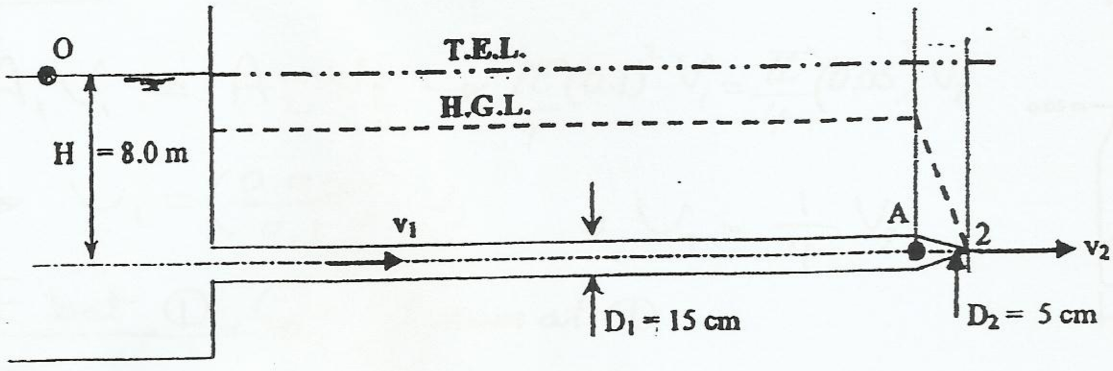
Ex. 5

Determine the discharge and the pressure at A, H = 8.0 m.

(a) For ideal fluid flow.

(b) If the losses up to section A are $\frac{4v_1^2}{2g}$ and the nozzle losses are $\frac{0.05v_2^2}{2g}$

Solution:



a) Case of No-losses

Apply B.E bet ⓞ & ② datum at ②

$8+0+0 = 0+0+\frac{V_2^2}{2g} \longrightarrow V_2 = 12.53 \text{ m/s}$

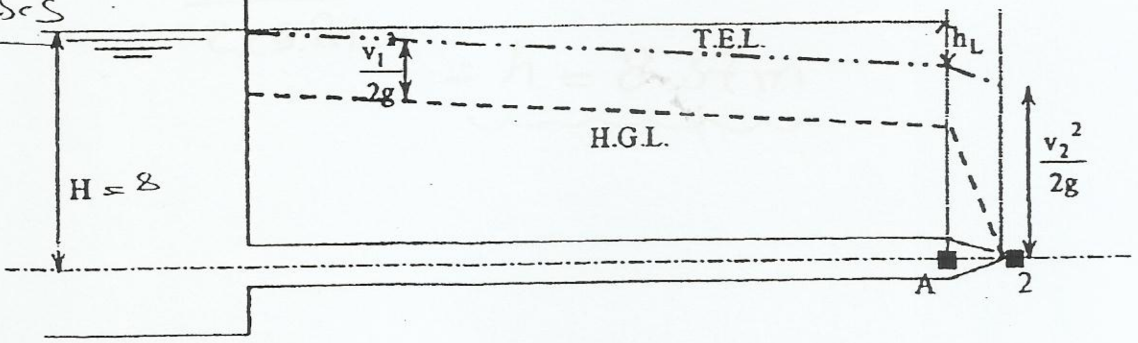
$\therefore Q = A_2V_2 = \frac{\pi(0.05)^2}{4} * 12.53 = 0.025 \text{ m}^3/\text{s}$

Apply B.E bet ⓞ & Ⓐ datum at Ⓐ

$8+0+0 = 0 + \frac{P_A}{\gamma} + \frac{V_A^2}{2g}$ where $V_A = V_2 * \left(\frac{D_2}{D_A}\right)^2 = 1.39 \text{ m/s}$

$\hookrightarrow P_A = 77513 \text{ Pa}$

b) Considering losses



Apply B.E bet ⓞ & ② datum at ②

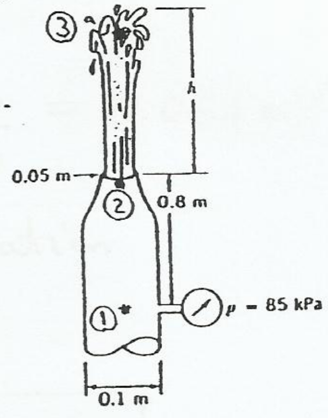
$8+0+0-\frac{4V_1^2}{2g}-\frac{0.05V_2^2}{2g} = 0+0+\frac{V_2^2}{2g}$ but $V_1 = 9V_2$ ← $\left(\frac{15}{5}\right)^2$

$\hookrightarrow \frac{1}{2g}\left[V_2^2 + 0.05V_2^2 + 4(9V_2)^2\right] = 8 \Longrightarrow V_2 = 11.94 \text{ m/s}$

$\hookrightarrow Q = 0.0234 \text{ m}^3/\text{s}$

EX: (6)

Water flows without viscous effects from the nozzle shown in the figure. **Determine** the flowrate and the height, h, to which the water can flow.



Solution:-

$$A_1 V_1 = A_2 V_2 \rightarrow \frac{\pi}{4}(0.1)^2 V_1 = \frac{\pi}{4}(0.05)^2 V_2$$

$$\rightarrow V_1 = \left(\frac{0.05}{0.1}\right)^2 V_2 \qquad \therefore V_1 = \frac{1}{4} V_2$$

B.E bet ①, ② & datum at ①

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + 0 = 0 + \frac{V_2^2}{2g} + 0.8$$

$$\frac{85\times10^3}{9810} + \frac{V_1^2}{2\times9.81} = \frac{(4V_1)^2}{2\times9.81} + 0.8 \Rightarrow V_1 = 3.21\ m/s$$

$$\& \ V_2 = 4V_1 = 12.82\ m/s$$

$$\therefore Q = A_1 V_1 = 0.025\ m^3/s$$

B.E bet ②, ③ datum at ②

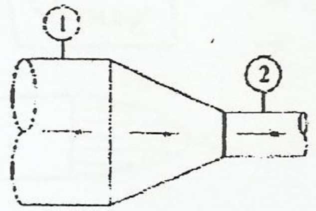
$$\underbrace{\frac{P_2}{\gamma}}_{0} + \frac{V_2^2}{2g} + \underbrace{Z_2}_{0} = \underbrace{\frac{P_3}{\gamma}}_{0} + \underbrace{\frac{V_3^2}{2g}}_{0} + Z_3$$

at the end of water Jet (h) → V = Zero

$$\rightarrow \frac{V_2^2}{2g} = h = \frac{(12.82)^2}{2\times9.81}$$

$$\therefore h = 8.37\ m$$

*7/2-1 Water at 22° C flows steadily at 600 N /s through the nozzle in Fig. If $D1$ = 16 cm and $D2$ = 4.5 cm, compute the average velocity, in m/s, at
(a) Section 1
(b) Section 2.



$Q_w = 600\ N/s$

$\rightarrow Q = \frac{Q_w}{\gamma_w} = \frac{600}{9810} = 0.061\ m^3/s$

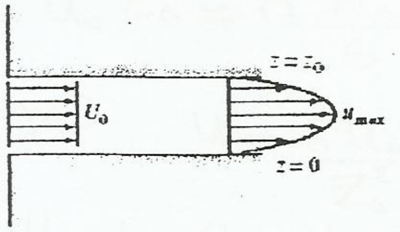
Applying Continuity Equation

$Q = A_1 V_1 = A_2 V_2$

$\rightarrow V_1 = \frac{Q}{A_1} = \frac{0.061}{\pi (0.16)^2/4} = \boxed{3.03\ m/s}$

$V_2 = \frac{Q}{A_2} = \frac{0.061}{\pi (0.045)^2/4} = \boxed{38.35\ m/s}$

7/2-2 Incompressible steady flow in the inlet between parallel plates in Fig. is uniform, $u = U0$ = 12 cm/s, while downstream the flow develops into the parabolic laminar profile $u = a z(z_o - z)$, where a is a constant. If z_o = 6 cm and the fluid is SAE 30 oil at 20°C, what is the value of u_{max} in cm/s?

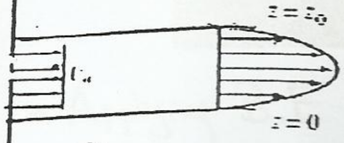


7/2-2 Incompressible steady flow in the inlet between parallel plates in Fig. is uniform, $u = U0 = 12$ cm/s, while downstream the flow develops into the parabolic laminar profile $u = a\,z(z_o - z)$, where a is a constant. If $z_o = 6$ cm and the fluid is SAE 30 oil at 20°C, what is the value of u_{max} in cm/s?

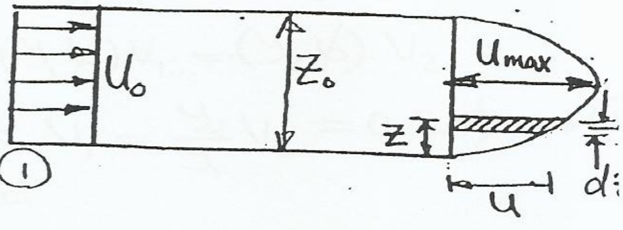
Req $\boxed{U_{max}}$

$U_o = 12$ cm/s

$Z_o = 6$ cm



$U = aZ(Z_o - Z) \rightarrow (1)$



* Consider a strip (dZ) with discharge (dq) & velocity (u)

$$dq = u * dA = aZ(Z_o - Z) * (dZ * 1.0) \quad \left\{ dA = dZ * 1.0^{cm} \right\}$$

$$\rightarrow dq = (aZZ_o - aZ^2)\,dZ$$

By Integration

$$q = \left[\frac{aZ_oZ^2}{2} - \frac{aZ^3}{3} \right]_{Z=0}^{Z=Z_o} \Rightarrow q = a\left[\frac{Z_o^3}{2} - \frac{Z_o^3}{3} \right] = a\frac{Z_o^3}{6}$$

But from uniform flow conditions

$$q = U_o Z_o$$

$$\therefore q = U_o Z_o = a\frac{Z_o^3}{6} \Rightarrow a = \frac{6U_o}{Z_o^2}$$

$\therefore$ In equ (1)

$$U = \frac{6U_o}{Z_o^2} Z(Z_o - Z) \quad \text{General equation} \rightarrow (2)$$

To get the location of U_{max} $\quad \frac{dU}{dZ} = 0$

$$\therefore \frac{6U_o}{Z_o^2}(Z_o - 2Z) = 0 \rightarrow Z_o - 2Z = 0 \quad \therefore Z = \frac{Z_o}{2}$$

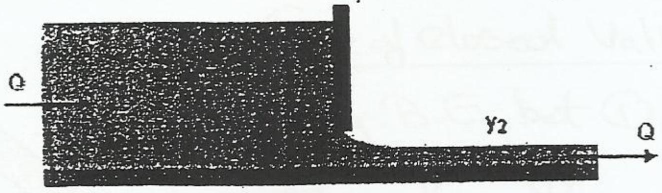
$\therefore U_{max}$ at $Z = \frac{Z_o}{2}$ (middle of spacing between the plates)

In equ (1)

$$U_{max} = \frac{6U_o}{Z_o^2} * \frac{Z_o}{2}\left(Z_o - \frac{Z_o}{2}\right) = \frac{6U_o}{Z_o^2} * \frac{Z_o^2}{4}$$

$$\rightarrow U_{max} = \frac{6}{4}U_o = \boxed{1.5U_o} \qquad \boxed{\therefore U_{max} = 18 \text{ cm/s}}$$

7/2-3 for flow under a sluice gate as illustrated in the figure, there is no significant loss. If y_1 = 1m and y_2 =0.20m. What are the velocities V_1 and V_2 at locations y_1 and y_2, respectively (assume a channel width of 1m).



* Applying Continuity Equation

$$A_1 V_1 = A_2 V_2$$

$$(y_1 \cancel{b}) V_1 = (y_2 \cancel{b}) V_2$$

$$\therefore V_1 = \frac{y_2}{y_1} V_2 = 0.2 V_2 \rightarrow (1)$$

* Applying B.E bet (1), (2) datum at bed

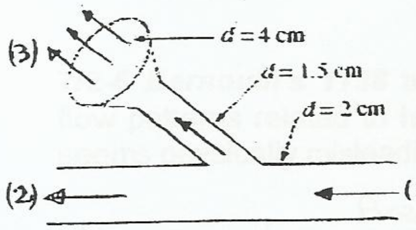
$$\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g} \quad \text{where } \frac{P}{\gamma} = y$$

$$\hookrightarrow y_1 + 0 + \frac{V_1^2}{2g} = y_2 + 0 + V_2^2/2g$$

From (1) $\rightarrow$ $$1 + \frac{(0.2 V_2)^2}{2 \times 9.81} = 0.2 + \frac{V_2^2}{2g} \longrightarrow \boxed{V_2 = 4.04 \text{ m/s}}$$

$$\rightarrow V_1 = 0.2 V_2 = \boxed{0.808 \text{ m/s}}$$

7/2-4 Water at 20°C flows steadily through the piping junction in Fig., entering section 1 with 30 gal / min. The average velocity at section 2 is 2.9 m/s. A portion of the flow is diverted through the showerhead, which contains 76 holes of 1.2mm diameter. Assuming uniform shower flow, estimate the exit velocity from the showerhead jets.



Given $Q_1 = 30$ gal/min $[1 \text{ gal} = 3.78 \text{ lit}]$

$V_2 = 2.9$ m/s

$$A_3 = 76 \times \frac{\pi}{4} (1.2 \times 10^{-3})^2 = 8.59 \times 10^{-5} \text{ m}^2$$

Sol

$$Q_1 = 20 \frac{\text{gal}}{\text{min}} = \frac{20 \times 3.78 \times 10^{-3}}{60} = 1.26 \times 10^{-3} \text{ m}^3/\text{s}$$

$$Q_2 = A_2 V_2 = \frac{\pi (0.02)^2}{4} \times 2.9 = 9.11 \times 10^{-4} \text{ m}^3/\text{s}$$

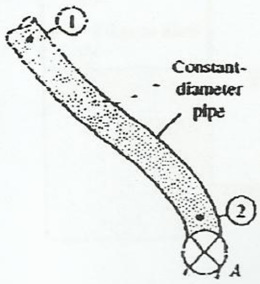
By Continuity Equation

$$Q_1 = Q_2 + Q_3$$

$$\hookrightarrow Q_3 = Q_1 - Q_2 = 3.49 \times 10^{-4} \text{ m}^3/\text{s}$$

$$= A_3 V_3 \longrightarrow V_3 = \frac{Q_3}{A_3} = \boxed{4.06 \text{ m/s}}$$

7/2-5 The long pipe in Fig. is filled with water at 10°C. When valve *A* is closed, $p_2 - p_1 =$ 65 k Pa. When the valve is open and water flows at 450 m^3 /h, $p_1 - p_2 =$ 180 k Pa. What is the friction head loss between 1 and 2, in m, for the flowing condition?



* Case of closed Valve [Static Conditions]

Apply B.E bet ①,② datum at ②

$$Z_1 + \frac{P_1}{\gamma} + \cancel{\frac{V_1^2}{2g}} = \cancel{Z_2} + \frac{P_2}{\gamma} + \cancel{\frac{V_2^2}{2g}}$$

$$\hookrightarrow Z_1 = \frac{P_2 - P_1}{\gamma} = \frac{65\times10^3}{9810} = \underline{6.63\,m}$$

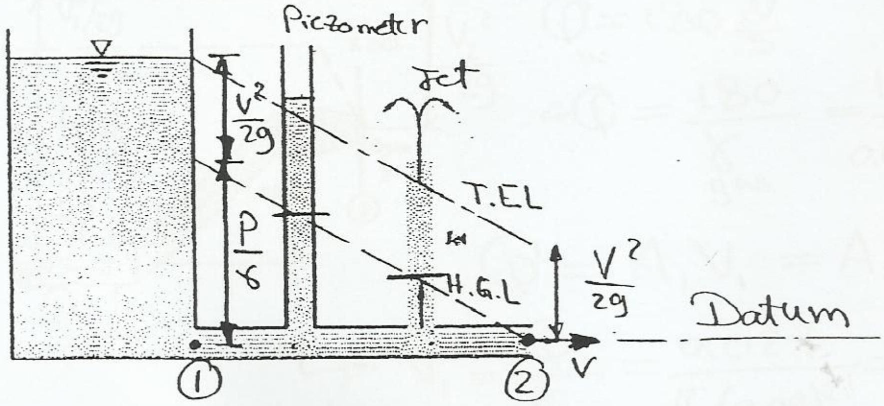
* Case of open Valve

B.E bet ①,② datum at ② $[H_1 - h_L = H_2]$

$$Z_1 + \frac{P_1}{\gamma} + \cancel{\frac{V_1^2}{2g}} - h_L = \cancel{Z_2} + \frac{P_2}{\gamma} + \cancel{\frac{V_2^2}{2g}}$$ as $d_1 = d_2$ ∴ $V_1 = V_2$

$$\hookrightarrow h_L = Z_1 + \frac{P_1 - P_2}{\gamma} = 6.63 + \frac{180\times10^3}{9810} \Rightarrow \boxed{h_L = 24.98\,m}$$

7/2-6 ***Bernoulli's 1738*** treatise *Hydrodynamica* contains many excellent sketches of flow patterns related to his frictionless relation. One, however, redrawn here as Fig., seems physically misleading. Can you explain what might be wrong with the figure?



* Consider point ①, ②

Total head at ① $= \frac{P}{\gamma} + \frac{V^2}{2g}$

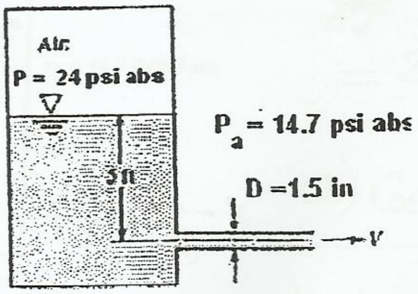
Total head at ② $= \frac{V^2}{2g}$

$Z_1 = Z_2 = 0$
$P_2 = \text{zero}$
$V_1 = V_2 = V$

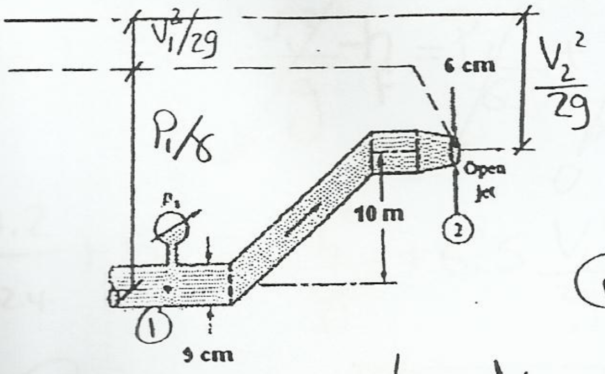
∴ $H_1 \neq H_2$ ∴ The flow can not be frictionless

* Water level in the piezometer & at the top of the Jet must not exceed the H.G.L

7/2-7 The liquid in Fig. is Gasoline S.G. = 0.72. Estimate the flow rate from the tank for (*a*) no losses and (*b*) pipe losses $h_f = 6.5V^2/(2g)$.



7/2-8 In Fig. the fluid is gasoline at 20°C (S.G. 0.68) at a weight flux of 180 N/s, assuming no losses estimate the gage pressure at section 1.



$Q_w = 180 \frac{N}{s}$

$\therefore Q = \frac{180}{\gamma_{gas}} = \frac{180}{0.68 \times 9810} = 0.027 \, m^3/s$

$Q = A_1 V_1 = A_2 V_2$

$\hookrightarrow V_1 = \frac{Q}{A_1} = \frac{0.027}{\frac{\pi}{4}(0.09)^2} = 4.24 \, m/s$

$V_2 = \frac{Q}{A_2} = \frac{0.027}{\frac{\pi}{4}(0.06)^2} = 9.55 \, m/s$

Apply B.E bet ①, ② datum at ①

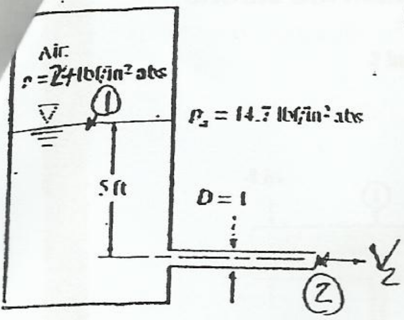
$\cancel{Z_1} + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \cancel{Z_2} + \cancel{\frac{P_2}{\gamma}} + \frac{V_2^2}{2g}$

$\quad 0 \qquad\qquad\qquad\qquad 10 \quad 0$

$\hookrightarrow \frac{P_1}{\gamma_{gas}} = Z_2 + \frac{V_2^2 - V_1^2}{2g} = 13.73 \Rightarrow P_1 = 13.73 * (0.68 \times 9810)$

$= \boxed{91604 \, Pa}$

-7. The liquid in Fig. is Gasoline S.G. =0.72 Estimate the flow rate from the tank for (a) no losses and (b) pipe losses $h_f = 6.5V^2/(2g)$.



$$(P_1)_{gage} = (P_1)_{abs} - P_{atm}$$

$$= 24 - 14.7 = 9.3\ lb/in^2 \rightarrow P_1 = 9.3 \times 14.4 = 1339.2\ lb/ft^2$$

a) Case of No losses :-

Apply B.E bet ①,② datum at ②

$$\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g}$$

$$\frac{1339.2}{(0.72 * 62.4)} + 5 + 0 = 0 + 0 + \frac{V_2^2}{2g} \rightarrow V_2 = 47.35\ ft/s$$

$$\therefore Q = V_2 A_2 = \boxed{0.58\ ft^3/s} \qquad \left\{ A_2 = \frac{\pi}{4}\left(\frac{1.5}{12}\right)^2 = 12.3 \times 10^{-3}\ ft^2 \right.$$

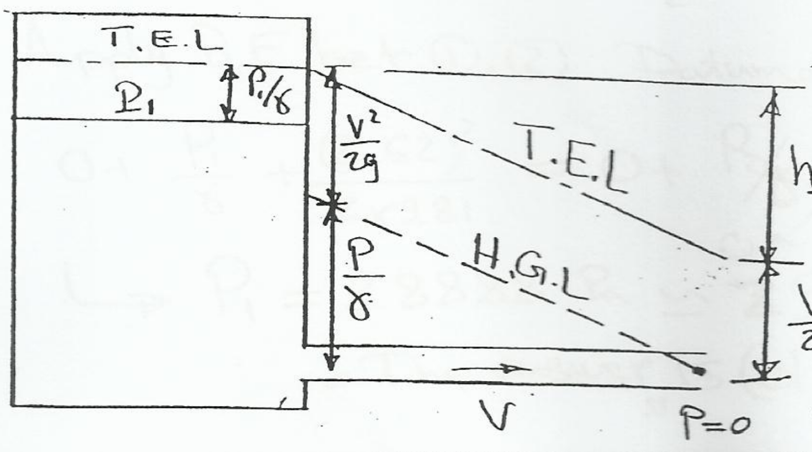
b) Case of pipe losses

B.E bet ①,②

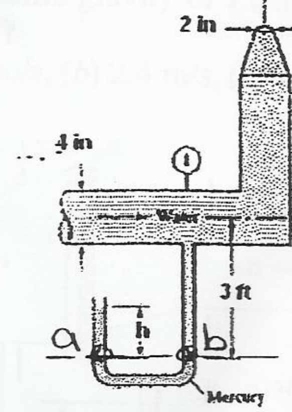
$$\frac{P_1}{\gamma} + Z_1 + \cancel{\frac{V_1^2}{2g}} - h_f = \cancel{\frac{P_2}{\gamma}} + \cancel{Z_2} + \frac{V_2^2}{2g}$$

$$\frac{1339.2}{0.72 \times 62.4} + 5 = \frac{V_2^2}{2g} + 6.5\frac{V_2^2}{2g} = 7.5\frac{V_2^2}{2g} \rightarrow V_2 = 17.29\ ft/s$$

$$\therefore Q = A_2 V_2 = \boxed{0.213\ ft^3/s}$$



7/2-9 In Fig. both fluids are at 20°C. If V_1 = 2.4 ft/s and losses are neglected, what should thé manometer reading *h* ft be?



$$Q = A_1 V_1 = A_2 V_2$$

$$\hookrightarrow V_2 = V_1 \frac{A_1}{A_2} = V_1 \frac{D_1^2}{D_2^2}$$

$$\therefore V_2 = 2.4\left(\frac{4}{2}\right)^2 = 9.6 \text{ ft/s}$$

Apply B.E bet ①,② Datum at ①

$$\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g}$$

$$\rightarrow \frac{P_1}{\gamma} + 0 + \frac{(2.4)^2}{2 \times 32.2} = 0 + 12 + \frac{(9.6)^2}{2 \times 32.2}$$

$$\hookrightarrow P_1 = 832.52 \text{ lb/ft}^2$$

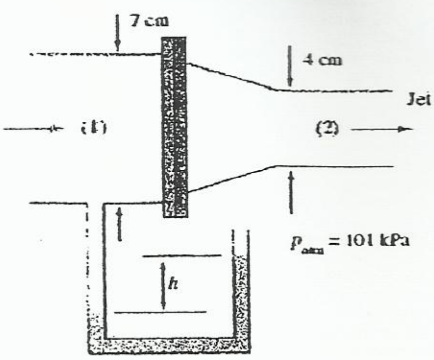
<u>In Manometer</u>

$$P_a = P_b$$

$$\gamma_{Hg} * h = P_1 + \gamma_w * 3$$

$$\rightarrow h = \frac{832.52 + 62.4 \times 3}{13.6 \times 62.4} = \boxed{1.2 \text{ ft}}$$

7/2-10 In Fig. water exits from a nozzle into atmospheric pressure of 101 k Pa. If the flow rate is 160gal/min and friction is neglected, what is the gage pressure at section 1? (*a*) 1.4 k Pa, (*b*) 32 k Pa, (c) 43 k Pa, (*d*) 29 k Pa, (*e*) 123 k Pa



$$Q = 160 \text{ gal/min}$$

$$= \frac{160 \times 3.78 \times 10^{-3}}{60} = 0.0101 \text{ m}^3/\text{s}$$

$$V_1 = \frac{Q}{A_1} = 2.62 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = 8.04 \text{ m/s}$$

Apply B.E bet ①,② Datum at Centerline

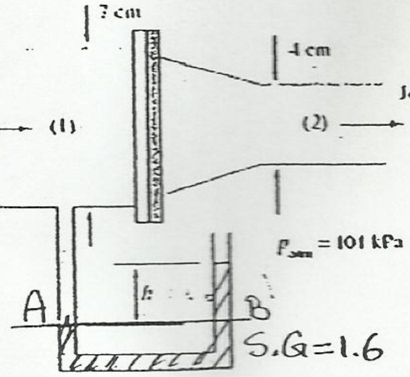
$$0 + \frac{P_1}{\gamma} + \frac{(2.62)^2}{2 \times 9.81} = 0 + \cancel{\frac{P_2}{\gamma}} + \frac{(8.04)^2}{2 \times 9.81}$$

0

$$\hookrightarrow P_1 = 28888 \text{ Pa} \simeq 29 \text{ KPa}$$

$\therefore$ The answer is (d)

7/2-1/ In Fig. water exits from a nozzle into atmospheric pressure of 101 kPa. If the manometer fluid has a specific gravity of 1.6 and h = 66 cm, with friction neglected, what is the average velocity at section 2?
(a) 4.55 m/s, (b) 2.4 m/s, (c) 2.95 m/s, (d) 5.55 m/s, (e) 3.4 m/s



From the manometer

$$P_A = P_B$$

$$P_1 + \gamma_w h = \gamma_m h$$

$$\hookrightarrow P_1 = h(\gamma_m - \gamma_w) = 0.66 \times 9810(1.6-1)$$

$$\therefore P_1 = 3884 \text{ Pa}$$

Apply B.E bet (1), (2)

$$0 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = 0 + 0 + \frac{V_2^2}{2g}$$

where $V_1 D_1^2 = V_2 D_2^2$

$$\therefore V_1 = \left(\frac{D_2}{D_1}\right)^2 V_2$$

$$V_1 = 0.327 V_2$$

$$\rightarrow \frac{3884}{9810} + \frac{(0.327)^2 V_2^2}{2 \times 9.81} = \frac{V_2^2}{2 \times 9.81}$$

$$\longrightarrow V_2 = 3.4 \text{ m/s} \text{ [Answer(e)]}$$